### Question 1:
A progression has a first term of 12 and a fifth term of 18.

(i) Find the sum of the first 25 terms if the progression is arithmetic. [3]

(ii) Find the 13th term if the progression is geometric. [4]

**Answer:**

(i) \[ a = 12, \ qquad a + 4d = 18 \implies d = 1.5 \]

\[ S_{25} = 25/2(24 + 24 \times 1.5) = 750 \]

(ii) \[ a = 12, \quad r^4 = 18, \quad r = 1.5 \]

13th term = \[ a \cdot r^{12} = 12 \times (1.5)^{12} \]

\[ = 40.5 \text{ or } 40.6 \]

<table>
<thead>
<tr>
<th></th>
<th>B1</th>
<th>A1</th>
<th>Correct only</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>M1</td>
<td></td>
<td>Use of ( S_n ) formula.</td>
</tr>
<tr>
<td></td>
<td>M1 A1</td>
<td>Correct only.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>M1</td>
<td>Correct method for ( r ) or ( r^4 ) (needs ( a )).</td>
<td></td>
</tr>
<tr>
<td></td>
<td>A1 4</td>
<td>Correct only.</td>
<td></td>
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</tbody>
</table>

### Question 2:
A geometric progression, for which the common ratio is positive, has a second term of 18 and a fourth term of 8. Find

(i) the first term and the common ratio of the progression. [3]

(ii) the sum to infinity of the progression. [2]

**Answer**

\[ a = 18, \quad a \cdot r^2 = 8 \]

Solution to give \( r = 2/3 \)

\[ a = 18 \cdot r = 27.0 \]

(ii) Sum to infinity = \( a/(1-r) \)

Answer = 81.0

<table>
<thead>
<tr>
<th></th>
<th>M1</th>
<th>DM1</th>
<th>Correct method on correct 2 equations. Any 2 equations of type ( ar^n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A1 3</td>
<td>Correct method on correct 2 equations. For his ( 18 + r )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>M1</td>
<td>A1√2</td>
<td>Correct formula applied -- even if ( r &lt; 1 ). Follow through provided ( r &lt; 1 ).</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(ignore ( r = 2/3 ))</td>
<td></td>
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</tbody>
</table>

### Question 3

(a) A debt of $3726 is repaid by weekly payments which are in arithmetic progression. The first payment is $60 and the debt is fully repaid after 48 weeks. Find the third payment. [3]

(b) Find the sum to infinity of the geometric progression whose first term is 8 and whose second term is 4. [3]

**Answer**

\[ a = 60, \quad n = 48, \quad S_n = 3726 \]

\[ S_n \text{ formula used} \]

\[ d = 50.75 \]

3rd term = \( a + 2d = 61.50 \)

(b) \[ a = 6, \quad ar = 4 \implies r = 3/2 \]

\[ S_n = a/(1-r) = 18 \]

<table>
<thead>
<tr>
<th></th>
<th>M1</th>
<th>A1</th>
<th>Correct formula (M0 if nth term used)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A1</td>
<td>A1√</td>
<td>Co</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Use of ( a+2d ) with his ( d ). 61.5 ok.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>M1</td>
<td>M1A1</td>
<td>( a, \ ar \text{ correct, and } r \text{ evaluated} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Correct formula used, but needs ( r &lt; 1 ) for M mark</td>
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</tbody>
</table>

### Question 4:
Find

(i) the sum of the first ten terms of the geometric progression 81, 54, 36, ... . [3]

(ii) the sum of all the terms in the arithmetic progression 180, 175, 170, ..., 25. [3]

**Answer**
Question 5:
A geometric progression has first term 64 and sum to infinity 256. Find

(i) the common ratio.  
(ii) the sum of the first ten terms.

**Answer**

(i) \( a(1-r) = 256 \) and \( a = 64 \)
\[
\rightarrow r = \frac{3}{4}
\]

(ii) \( S_{10} = 64(1-0.75^{10}) \) \( 1-0.75 \)
\[
\rightarrow S_{10} = 242
\]

Question 6:
A geometric progression has 6 terms. The first term is 192 and the common ratio is 1.5. An arithmetic progression has 21 terms and common difference 1.5. Given that the sum of all the terms in the geometric progression is equal to the sum of all the terms in the arithmetic progression, find the first term and the last term of the arithmetic progression.

**Answer**

G.P. \( a = 192, r = 1.5, n = 6 \)
A.P. \( a = a, d = 1.5, n = 21 \)
\[
S_6 \text{ for GP } = 192(1.5^6 - 1) \div 0.5 = 3090
\]
\[
S_{21} \text{ for AP } = \frac{21}{2} (2a + 20 \times 1.5)
\]
\[
\text{Equate and solve } \rightarrow a = 175
\]
\[
21^{st} \text{ term in AP } = a + 20d = 205 \text{ for from } 3090 = 21(a + 1.5)/2
\]

Question 7:
A small trading company made a profit of $250,000 in the year 2000. The company considered two different plans, plan A and plan B, for increasing its profits.

Under plan A, the annual profit would increase each year by 5% of its value in the preceding year. Find, for plan A,

(i) the profit for the year 2008,

(ii) the total profit for the 10 years 2000 to 2009 inclusive.

Under plan B, the annual profit would increase each year by a constant amount $D$.

(iii) Find the value of $D$ for which the total profit for the 10 years 2000 to 2009 inclusive would be the same for both plans.

**Answer**
(i) GP with $a = 250000$
   $r = 1.05$
   Year 2008 is the 9th term
   $a^9 = 250000 	imes 1.05^9 = 369000$
   B1
   M1
   A1
   [3]

   For any use of $r = 1.05$
   $(25000 + 0.05 	imes 25000$ gets B1)
   Use of $a^9$ with $r \approx 8$ or 9
   Answer rounding to 369 000, ft on $r$.

(ii) $S_{10} = 250000(1.05^{10} - 1): 0.05$
     $= 3140000$
     M1
     A1
     [2]

   Use of correct $S_n$ formula – for 10 only
   Co – must round to the correct answer
   (adds 10 numbers correctly M1 A1)

(iii) AP
   $S_n = \frac{5(500000 + 9D)}{2}$
   = answer to (ii)
   $\rightarrow D = 14300$
   M1
   DM1
   A1
   [3]

Correct $S_n$ formula.
Forming + soln
Co.

\[9709/01/O/N/05\]

**Question 8:**
(a) Find the sum of all the integers between 100 and 400 that are divisible by 7.

(b) The first three terms in a geometric progression are 144, $x$ and 64 respectively, where $x$ is positive.
   Find
   (i) the value of $x$,
   (ii) the sum to infinity of the progression.

**Answer**

(i) $a = 105$
   Either $f = 399$ or $d = 7$
   $n = 43$
   $\rightarrow 10380$
   B1
   B1
   B1
   B1
   co
   [4]

(ii) $r^2 = 64/144 \rightarrow r = \frac{2}{3}$
   M1
   award in either part

   (i) Either $x = ar \rightarrow x = 96$
      or $\frac{144}{x} = \frac{96}{64} \rightarrow x = 96$
      M1 A1
      either method ck

   (ii) $S_n = \frac{a}{1 - r}$
      $\rightarrow 432$
      M1
      A1
      Used with his $a$ and $r$
      [5]

\[9709/01/O/N/06\]

**Question 9:**
Each year a company gives a grant to a charity. The amount given each year increases by 5% of its value in the preceding year. The grant in 2001 was $5000. Find
(i) the grant given in 2011,

(ii) the total amount of money given to the charity during the years 2001 to 2011 inclusive.

**Answer**

(i) $r = 1.05$ with GP
   2011 is 11 years.
   Uses $a^r$
   $\rightarrow 5000 \times 1.05^{10}$
   B1
   M1
   A1
   [3]

   Allow if correct formula with $n = 10$
   co. (allow 3 sf)

(ii) Use of $S_n$ formula
   $\rightarrow 5000(1.05^{11} - 1) / 0.05$
   M1
   A1
   [2]

   Allow if used correctly with 10 or 11.
   co (or 50000)

\[9709/01/M/J/06\]
Question 10:
The 1st term of an arithmetic progression is $a$ and the common difference is $d$, where $d \neq 0$.

(i) Write down expressions, in terms of $a$ and $d$, for the 5th term and the 15th term. [1]

The 1st term, the 5th term and the 15th term of the arithmetic progression are the first three terms of a geometric progression.

(ii) Show that $3a = 8d$. [3]

(iii) Find the common ratio of the geometric progression. [2]

Answer

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<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>i. (i) $a + 4d$ and $a + 14d$</td>
<td>B1</td>
<td>Both correct.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>M1</td>
<td>Correct first step – award the mark for both of these starts.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>M1</td>
<td>Correct elimination of $r$. co.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A1</td>
<td>nb answer was given.</td>
</tr>
<tr>
<td>(ii) $a + 4d = ar$, $a + 14d = ar^2$</td>
<td></td>
<td>M1 A1</td>
<td>Statement + some substitution. co.</td>
</tr>
<tr>
<td>or $\frac{a}{a + 4d} = \frac{a + 4d}{a + 14d}$ or $ac = b^2$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rightarrow 3a = 8d$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(iii) $r = \frac{a + 4d}{a}$ or $\frac{a + 14d}{a + 4d} = 2.5$</td>
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</table>

Question 11:
The second term of a geometric progression is 3 and the sum to infinity is 12.

(i) Find the first term of the progression. [4]

An arithmetic progression has the same first and second terms as the geometric progression.

(ii) Find the sum of the first 20 terms of the arithmetic progression. [3]

Answer

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<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>(i) $a=3$ and $\frac{a}{1-r} = 12$</td>
<td>B1 B1</td>
<td>co for each one.</td>
<td></td>
</tr>
<tr>
<td>Solution of sim eqns $\rightarrow a = 6$</td>
<td>M1 A1</td>
<td>Needs to eliminate $a$ or $r$ correctly. co (M mark needs a quadratic)</td>
<td></td>
</tr>
<tr>
<td>(ii) $a=6$, $d=-3$</td>
<td>B1 A1</td>
<td>For $d = 3$ – his “6”.</td>
<td></td>
</tr>
<tr>
<td>$S_{20} = 10(12 - 57)$</td>
<td>M1</td>
<td>Sum formula must be correct and used. co.</td>
<td></td>
</tr>
<tr>
<td>$\rightarrow -450$</td>
<td>A1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Question 12:
The first term of a geometric progression is 81 and the fourth term is 24. Find

(i) the common ratio of the progression, [2]

(ii) the sum to infinity of the progression. [2]

The second and third terms of this geometric progression are the first and fourth terms respectively of an arithmetic progression.

(iii) Find the sum of the first ten terms of the arithmetic progression. [3]

Answer
Question 13:
The first term of an arithmetic progression is 6 and the fifth term is 12. The progression has \( n \) terms and the sum of all the terms is 90. Find the value of \( n \). [4]

**Answer**

| 3rd term \( a_3 = a + 2d = 12 \) | \( d = 1.5 \) | \( S_5 = \frac{n}{2} (12 + (n - 1)1.5) = 90 \) | \( n^2 + 7n - 120 = 0 \) | \( n = 8 \) | B1 | Correct value of \( d \) | M1 | Use of correct formula with his \( d \) | DM1 | Correct method for soln of quadratic | A1 | Co (ignore inclusion of \( n = -15 \) | [4] |

Question 14:
A progression has a second term of 96 and a fourth term of 54. Find the first term of the progression in each of the following cases:

(i) the progression is arithmetic, [3]

(ii) the progression is geometric with a positive common ratio. [3]

**Answer**

(i) \( a + d = 96 \) and \( a + 3d = 54 \)

\[ d = -21 \quad a = 117 \]

B1 | M1 | A1 | For both expressions. Correct method of solution. Co (no working, \( d \) correct, \( a \) wrong 0/3) [3]

(ii) \( ar = 96 \) and \( ar^3 = 54 \)

\[ r^2 = \frac{54}{96} \quad r = \frac{3}{4} \]

\[ a = 128 \]

B1 | M1 | A1 | For both expressions. Correct method of solution. Co. \( r = \pm \frac{3}{4} \), no penalty. [3]

Question 15:
The first term of an arithmetic progression is 8 and the common difference is \( d \), where \( d \neq 0 \). The first term, the fifth term and the eighth term of this arithmetic progression are the first term, the second term and the third term, respectively, of a geometric progression whose common ratio is \( r \).

(i) Write down two equations connecting \( d \) and \( r \). Hence show that \( r = \frac{3}{4} \) and find the value of \( d \). [6]

(ii) Find the sum to infinity of the geometric progression. [2]

(iii) Find the sum of the first 8 terms of the arithmetic progression. [2]

**Answer**
Question 16:
(a) Find the sum to infinity of the geometric progression with first three terms 0.5, 0.5² and 0.5³.

(b) The first two terms in an arithmetic progression are 5 and 9. The last term in the progression is the only term which is greater than 200. Find the sum of all the terms in the progression.

Answer
(a) \( a = 0.5, \ r = 0.5^2 \)
   - Uses correct formula \( S_n = \frac{a}{1-r} \) (or 0.667)
   - \( S_n = \frac{1}{2} \) for both \( a \) and \( r \) uses correct formula with some \( a, r \).

(b) \( a = 5, \ d = 4 \)
   - Uses 200 = a + (n-1)d or T.I.
   - 50 terms in the progression
   - Use of correct Sum formula
   - \( S_n = 5150 \)

Question 17:
(a) The fifth term of an arithmetic progression is 18 and the sum of the first 5 terms is 75. Find the first term and the common difference.

(b) The first term of a geometric progression is 16 and the fourth term is \( \frac{27}{4} \). Find the sum to infinity of the progression.

Answer
(a) \( a + 4d = 18 \)
   - \( \frac{5}{2} (2a + 4d) = 75 \)
   - Solution
   - \( a = 12, \ d = 1\frac{1}{2} \)

(b) \( a = 16 \) and \( ar^3 = \frac{27}{4} \)
   - \( r = \frac{3}{4} \)
   - Sum to infinity = 64

Question 18:
The ninth term of an arithmetic progression is 22 and the sum of the first 4 terms is 49.

(i) Find the first term of the progression and the common difference. \[4\]

The \(n\)th term of the progression is 46.

(ii) Find the value of \(n\). \[2\]

Answer

\[
\begin{array}{|c|c|c|}
\hline
\text{9th term} & 22, S_4 = 49 & B1 \quad \text{co} \\
\hline
\text{f) } a + 8d = 22 & B1 \quad \text{co} \\
2(2a + 3d) = 49 & M1 \quad \text{Solution of two linear sim eqns. co} \\
\text{So final eqns} & \text{to solve, co.} \\
\rightarrow d = 1.5, a = 10 & [4] \\
\hline
\text{ii) } a + (n-1)d = 46 & M1 \quad \text{Correct formula needed and attempt to solve. co.} \\
& A1 \quad [2] \\
\rightarrow n = 25 & \text{9709/11/M/J/10} \\
\hline
\end{array}
\]

Question 19:

(a) Find the sum of all the multiples of 5 between 100 and 300 inclusive. \[3\]

(b) A geometric progression has a common ratio of \(-\frac{3}{4}\) and the sum of the first 3 terms is 35. Find

(i) the first term of the progression, \[3\]

(ii) the sum to infinity. \[2\]

Answer

\[
\begin{array}{|c|c|c|}
\hline
& a = 100, d = 5, & B1 \quad \text{Use of correct sum formula. co} \\
& n = 41 & M1 \quad [3] \\
& \rightarrow S = 8200 & \text{Use of correct sum formula. co} \\
\hline
\text{b) i) } a + ar + ar^2 & B1 \quad \text{co} \\
& = a(1 + r + r^2) & M1 \quad \text{Solution of equation. co} \\
& = 35 \rightarrow a = 45 & [3] \\
\hline
\text{ii) } S_n = \frac{a}{1-r} & M1 \quad \text{Correct use of formula. } \checkmark \text{ for his a} \\
& = 27 & A1 \quad [2] \\
\hline
\end{array}
\]

Question 20:

The first term of a geometric progression is 12 and the second term is \(-6\). Find

(i) the tenth term of the progression, \[3\]

(ii) the sum to infinity. \[2\]

Answer

\[
\begin{array}{|c|c|c|}
\hline
\text{f) } a = 12, ar = -6 & M1 \quad \text{Attempt at } r \text{ from “ar”} \\
r^9 = \frac{-3}{128} & M1 \quad \text{ar}^9 \text{ must be correct. co} \\
\hline
\text{ii) } S_n = \frac{a}{1-r} & M1 \quad \text{Correct formula used. } M1 \text{ needs } |r| < 1 \\
& \text{used } \rightarrow 8 & [2] \\
\hline
\end{array}
\]

Question 21:

(a) A geometric progression has first term 100 and sum to infinity 2000. Find the second term. \[3\]

(b) An arithmetic progression has third term 90 and fifth term 80.
Answer

(i) Find the first term and the common difference. [2]

(ii) Find the value of $m$ given that the sum of the first $m$ terms is equal to the sum of the first $(m + 1)$ terms. [2]

(iii) Find the value of $n$ given that the sum of the first $n$ terms is zero. [2]

Question 22:
(a) The first and second terms of an arithmetic progression are 161 and 154 respectively. The sum of the first $m$ terms is zero. Find the value of $m$. [3]

(b) A geometric progression, in which all the terms are positive, has common ratio $r$. The sum of the first $n$ terms is less than 90% of the sum to infinity. Show that $r^n > 0.1$. [3]

Question 23:
(a) The sixth term of an arithmetic progression is 23 and the sum of the first ten terms is 200. Find the seventh term. [4]

(b) A geometric progression has first term 1 and common ratio $r$. A second geometric progression has first term 4 and common ratio $\frac{1}{4} r$. The two progressions have the same sum to infinity, $S$. Find the values of $r$ and $S$. [3]
Question 24:
(a) A geometric progression has a third term of 20 and a sum to infinity which is three times the first term. Find the first term. \[4\]

(b) An arithmetic progression is such that the eighth term is three times the third term. Show that the sum of the first eight terms is four times the sum of the first four terms. \[4\]

Answer

(a) \[a + 7d = 3(a + 2d)\]  
\[
\Rightarrow 2a = d
\]
\[S_8 = 4(2a + 7d) = 32d \text{ or } 64a\]
\[S_4 = 2(2a + 3d) = 8d \text{ or } 16a\]

(b) \[a + 7d = 2a = d\]  
\[S_8 = 4(2a + 7d) = 32d \text{ or } 64a\]
\[S_4 = 2(2a + 3d) = 8d \text{ or } 16a\]

Question 25:
(a) A circle is divided into 6 sectors in such a way that the angles of the sectors are in arithmetic progression. The angle of the largest sector is 4 times the angle of the smallest sector. Given that the radius of the circle is 5 cm, find the perimeter of the smallest sector. \[6\]

(b) The first, second and third terms of a geometric progression are \(2k + 3\), \(k + 6\) and \(k\), respectively. Given that all the terms of the geometric progression are positive, calculate

(i) the value of the constant \(k\),

(ii) the sum to infinity of the progression. \[3\]

Answer

(a) \[a + 5d = 4a\] or \[\frac{(a + 4a) \times 6}{2}\]
\[\frac{6}{2}(2a + 5d) \text{ or } \frac{(a + 4a) \times 6}{2} = 360\]
\[\text{Sim Eqns } a = 24 \text{ or } \frac{2\pi}{15} \text{ radians}\]
\[\text{Arc length } = 50\]
\[\text{Perimeter } = 12.1\]

(b) \(\frac{k + 6}{2k + 3} = \frac{k}{k + 6}\)
\[\Rightarrow k^2 - 9k - 36 = 0 \Rightarrow k = 12\]
\[\text{(NB stating } a, ar, ar^2 \text{ as } f(k) \text{ gets M1)}\]

(ii) \[r = \frac{3}{2}, \quad a = 27\]
\[S_\infty = 27 + \frac{27}{r} = 81\]

Question 26:
A television quiz show takes place every day. On day 1 the prize money is $1000. If this is not won the prize money is increased for day 2. The prize money is increased in a similar way every day until it is won. The television company considered the following two different models for increasing the prize money.

Model 1: Increase the prize money by $1000 each day.

Model 2: Increase the prize money by 10% each day.
On each day that the prize money is not won the television company makes a donation to charity. The amount donated is 5% of the value of the prize on that day. After 40 days the prize money has still not been won. Calculate the total amount donated to charity.

(i) if Model 1 is used, [4]
(ii) if Model 2 is used. [3]

**Answer**

(i) $1000, 2000, 3000...$ or $50, 100, 150...$

\[
\frac{40}{2(1000 + 4000)} \quad or \quad \frac{40}{2(2000 + 3900)}
\]

\[
\times 5\% \text{ of attempt at valid sum} \quad 41000
\]

(ii) $1000, 1000 \times 1.1, 1000 \times 1.1^2 + ...$ or with $a = 50$

\[
\frac{1000(1.1^{10} - 1)}{1.1 - 1} = \frac{22100}{10}
\]

**Question 27:**

(a) An arithmetic progression contains 25 terms and the first term is $-15$. The sum of all the terms in the progression is 525. Calculate

(i) the common difference of the progression, [2]
(ii) the last term in the progression, [2]
(iii) the sum of all the positive terms in the progression. [2]

(b) A college agrees a sponsorship deal in which grants will be received each year for sports equipment. This grant will be $4000 in 2012 and will increase by 5% each year. Calculate

(i) the value of the grant in 2022, [2]
(ii) the total amount the college will receive in the years 2012 to 2022 inclusive. [2]

**Answer**

(a) $a = -15, \quad n = 25$

(i) Use of $S_n \rightarrow d = 3.$ [M1 A1 2]

(ii) Last term $= a + 24d$

\[
57
\]

(iii) Positive terms are 3, 6, ..., 57

Either $a = 0$ or 3, $n = 19$ or 20

Use of $S_{19}$ or $S_{20}$

\[
570
\]

(b) $r = 1.05$

(i) $11^{th} \text{ term } = ar^{10} = 6516 \text{ or } 6520$

(ii) $S_{11} = \frac{4000 \times (1.05^{11} - 1)}{0.05}$

\[
= 56800 \text{ or } 56827
\]

**Question 28:**

The first and second terms of a progression are 4 and 8 respectively. Find the sum of the first 10 terms given that the progression is

(i) an arithmetic progression, [2]
(ii) a geometric progression. [2]
Answer

(i) \[ \frac{5(8 + 9 \times 4)}{220} \]
\[ \frac{4(2^{10} - 1)}{2 - 1} \]
\[ 4092 \]

M1 Use correct formula with \( a=4, d=4 \)
A1 \[ \text{[2]} \]

(ii) M1 Use correct formula with \( a=4, r=2 \) or \( \frac{1}{2} \)
A1 \[ 4090 \text{ without } 4092 \text{ A0} \]

[2]

Question 29:
(a) The first two terms of an arithmetic progression are 1 and \( \cos^2 x \) respectively. Show that the sum of the first ten terms can be expressed in the form \( a - b \sin^2 x \), where \( a \) and \( b \) are constants to be found. \[ \text{[3]} \]

(b) The first two terms of a geometric progression are 1 and \( \frac{1}{3} \tan^2 \theta \) respectively, where \( 0 < \theta < \frac{1}{2} \pi \).
(i) Find the set of values of \( \theta \) for which the progression is convergent. \[ \text{[2]} \]

(ii) Find the exact value of the sum to infinity when \( \theta = \frac{1}{6} \pi \). \[ \text{[2]} \]

Answer

(a) \( S_{10} = \frac{10}{2[2 + 9(\cos^2 x - 1)]} \)
\( S_{10} = 5[2 - 9 \sin^2 x] \)
\( S_{10} = 10 - 45 \sin^2 x \)

M1 Correct formula with \( d = \pm(\cos^2 x - 1) \)
M1 Use of \( c^2 + s^2 = 1 \) in a correct \( S_{10} \)
A1 Or \( a = 10, b = 45 \)

[3]

(b) (i) \( \theta \frac{1}{3} \tan^2 \theta < 1 \) \( \text{oe} \)
\( \theta < \frac{\pi}{3} \)

M1 Allow <
A1 \( \text{cao} \) Allow <

[2]

(ii) \( S_{\infty} = \frac{1}{1 - \frac{1}{3} \tan^2 \pi \frac{1}{6}} \)
\( S_{\infty} = \frac{9}{8} \text{ or } 1.125 \)

M1 \( \text{cao} \)

[2]

Question 30:
(a) In an arithmetic progression, the sum of the first \( n \) terms, denoted by \( S_n \), is given by
\[ S_n = n^2 + 8n. \]
Find the first term and the common difference. \[ \text{[3]} \]

(b) In a geometric progression, the second term is 9 less than the first term. The sum of the second and third terms is 30. Given that all the terms of the progression are positive, find the first term. \[ \text{[5]} \]

Answer

(a) \( S_n = n^2 + 8n. \)
\( S_1 = 9 \rightarrow a = 9 \)
\( S_2 = 20 \rightarrow a + d = 11 \rightarrow d = 2 \)
(or equating \( n^2 + 8n \) with \( S_n \) and comparing coefficients)

B1 M1 A1 \[ \text{co} \]
Realises that \( S_2 \) is \( a + (a + d). \) co

[3]
(b) $a - ar = 9$

$ar + ar^2 = 30$

Eliminates $a → 3r^2 + 13r - 10 = 0$

or $→ 2a^2 - 57a + 81 = 0$

→ $r = \frac{3}{4}$

→ $a = 27$

<table>
<thead>
<tr>
<th>B1</th>
<th>M1</th>
<th>A1</th>
</tr>
</thead>
<tbody>
<tr>
<td>co</td>
<td>Complete elimination of $r$ or $a$</td>
<td></td>
</tr>
<tr>
<td>co</td>
<td>Correct quadratic.</td>
<td></td>
</tr>
<tr>
<td>co</td>
<td>(condone 27 or 1.5)</td>
<td></td>
</tr>
</tbody>
</table>

**Question 31:**
The first term of an arithmetic progression is 12 and the sum of the first 9 terms is 135.

(i) Find the common difference of the progression. [2]

The first term, the ninth term and the $n$th term of this arithmetic progression are the first term, the second term and the third term respectively of a geometric progression.

(ii) Find the common ratio of the geometric progression and the value of $n$. [5]

**Answer**

(i) Uses $S_n$

$\frac{9}{2}(24 + 8d) = 135 → d = \frac{3}{4}$

<table>
<thead>
<tr>
<th>M1</th>
<th>A1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uses correct formula</td>
<td></td>
</tr>
</tbody>
</table>

(ii) 9th term of AP = 12 + 8×$\frac{3}{4}$ = 18

GP 1st term 12, 2nd term 18

Common ratio $r = 18 + 12 = 1$½

3rd term of GP = $ar^2 = 27$

nth term of AP = 12 + (n - 1)$\frac{3}{4}$

$12 + (a - 1)\frac{3}{4} = 27 → n = 21$

<table>
<thead>
<tr>
<th>B1</th>
<th>M1</th>
<th>M1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{d}$ on “$d$”</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uses “$ar$”</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uses $ar^2$ or “$ar$” × $r$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M1A1</td>
<td>Links AP with GP. co</td>
<td></td>
</tr>
</tbody>
</table>

**Question 32:**
The first term of a geometric progression is $5\frac{1}{2}$ and the fourth term is $2\frac{1}{4}$. Find

(i) the common ratio, [3]

(ii) the sum to infinity. [2]

**Answer**

(i) $\frac{1}{4} = \frac{1}{3} r^3$

$r^3 = \frac{9}{3} = \frac{27}{64}$

$r = \frac{3}{4}$ or 0.75

<table>
<thead>
<tr>
<th>M1A1</th>
<th>A1</th>
</tr>
</thead>
<tbody>
<tr>
<td>[3]</td>
<td></td>
</tr>
</tbody>
</table>

(ii) $S_\infty = \frac{5\frac{1}{2}}{1 - \frac{3}{4}} = \frac{61}{3}$ (or $21\frac{1}{3}$ or 21.3)

<table>
<thead>
<tr>
<th>M1A1</th>
<th>cao</th>
</tr>
</thead>
<tbody>
<tr>
<td>[2]</td>
<td></td>
</tr>
</tbody>
</table>

9709/13/O/N/12

**Question 33:**
The first term of an arithmetic progression is 61 and the second term is 57. The sum of the first $n$ terms is $n$. Find the value of the positive integer $n$. [4]

**Answer**

\[\frac{n}{2} [122 + (n - 1)(-4)]\]

$n = \frac{n}{2} [122 + (n - 1)(-4)]$

$2n(n - 31) = 0$

$n = 31$

<table>
<thead>
<tr>
<th>M1</th>
<th>A1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attempt sum formula with $a = 61$, $d = -4$</td>
<td></td>
</tr>
<tr>
<td>Equated to $n$ co</td>
<td></td>
</tr>
<tr>
<td>Attempt to solve. Accept div. by $n$ co</td>
<td></td>
</tr>
</tbody>
</table>
(a) In a geometric progression, all the terms are positive, the second term is 24 and the fourth term is $13\frac{1}{2}$. Find

(i) the first term, [3]

(ii) the sum to infinity of the progression. [2]

(b) A circle is divided into $n$ sectors in such a way that the angles of the sectors are in arithmetic progression. The smallest two angles are $3^\circ$ and $5^\circ$. Find the value of $n$. [4]

**Answer**

(a) (i) $ar = 24$, $ar^2 = 13\frac{1}{2}$

Eliminates $a$ (or $r$) $ightarrow r = \frac{3}{4}$

$\rightarrow a = 32$

B1  
M1  
A1  

Both needed

Method of Solution.

(ii) sum to infinity $= 32 \div \frac{3}{4} = 128$

M1A1\h

Correct formula used. \h on value of $r$

(b) $a = 3$, $d = 2$

$\frac{n}{2}(6 + (n-1)2) = 360$

$\rightarrow 2n^2 + 4n - 720 = 0$

$\rightarrow n = 18$

B1  
M1  
A1  

Correct value for $d$

Correct $S_n$ used. no need for 360 here.

Correct quadratic

9709/12/O/N/12

**Question 35:**

(a) In an arithmetic progression the sum of the first ten terms is 400 and the sum of the next ten terms is 1000. Find the common difference and the first term. [5]

(b) A geometric progression has first term $a$, common ratio $r$ and sum to infinity 6. A second geometric progression has first term $2a$, common ratio $r^2$ and sum to infinity 7. Find the values of $a$ and $r$. [5]

**Answer**

(a) $\frac{10}{2}(2a + 9d) = 400$ \oe \n
$\frac{20}{2}(2a + 19d) = 1400$ \OR \n
$\frac{10}{2}[a + (10d) + 9d] = 1000$

$d = 6$  
$a = 13$

B1  
M1A1A1

$\rightarrow 2a + 9d = 80$

$\rightarrow 2a + 19d = 140$ or $2a + 29d = 200$

Solve sim. eqns both from $S_n$ formulae

M1B1

B1  
M1  
A1

$\rightarrow 2a + 19d = 140$ or $2a + 29d = 200$

Substitute or divide

A1\h

Ignore any other solns for $r$ and $a$

9709/11/O/N/13

**Question 36:**

The third term of a geometric progression is $-108$ and the sixth term is 32. Find

(i) the common ratio, [3]

(ii) the first term, [1]

(iii) the sum to infinity. [2]

**Answer**

(a) $ar^2 = -108$

$\frac{2a}{1-r^2} = 7$

$12(1-r) = 7$ or $\frac{1-r^2}{1-r} = \frac{12}{7}$

$r = \frac{5}{7}$ or 0.714

$a = \frac{12}{7}$ or 1.71(4)

9709/11/O/N/13
Question 37:
(a) The first and last terms of an arithmetic progression are 12 and 48 respectively. The sum of the first four terms is 57. Find the number of terms in the progression.
(b) The third term of a geometric progression is four times the first term. The sum of the first six terms is \( k \) times the first term. Find the possible values of \( k \).

Answer
(a) \( 57 = 2(24 + 3d) \rightarrow d = 1.5 \)
\[ 48 = 12 + (n - 1)1.5 \rightarrow n = 25 \]
M1 A1 M1 A1 Use of correct \( S_n \) formula.
Use of correct \( T_n \) formula.
(b) \( a(r^5 - 1) = ka \)
\[ \frac{r^5 - 1}{r - 1} = k \]
\[ k = 63 \quad \text{or} \quad k = -21 \]
B1 B1 B1 B1 (allow for \( r = 2 \))

Question 38:
(a) In a geometric progression, the sum to infinity is equal to eight times the first term. Find the common ratio.
(b) In an arithmetic progression, the fifth term is 197 and the sum of the first ten terms is 2040. Find the common difference.

Answer
(a) \[ \frac{a}{1 - r} = 8a \Rightarrow l(a) = 8(a)(1 - r) \]
\[ r = \frac{7}{8} \] B1 B1 [2]
(b) \[ a + 4d = 197 \]
\[ \frac{10}{2}[2a + 9d] = 2040 \]
\[ d = 14 \] B1 M1 A1 Or 2\(a + 9d = 408 \)
Attempt to solve simultaneously [4]

Question 39:
(a) In an arithmetic progression, the sum, \( S_n \), of the first \( n \) terms is given by \( S_n = 2n^2 + 8n \). Find the first term and the common difference of the progression.
(b) The first 2 terms of a geometric progression are 64 and 48 respectively. The first 3 terms of the geometric progression are also the 1st term, the 9th term and the \( n \)th term respectively of an arithmetic progression. Find the value of \( n \).

Answer
Question 40:
(a) An athlete runs the first mile of a marathon in 5 minutes. His speed reduces in such a way that each mile takes 12 seconds longer than the preceding mile.

(i) Given that the n<sup>th</sup> mile takes 9 minutes, find the value of n.
(ii) Assuming that the length of the marathon is 26 miles, find the total time, in hours and minutes, to complete the marathon.

(b) The second and third terms of a geometric progression are 48 and 32 respectively. Find the sum to infinity of the progression.

Answer

<table>
<thead>
<tr>
<th>(a)</th>
<th>(i)</th>
<th>(a = 300, d = 12)</th>
<th>M1 A1</th>
<th>[2]</th>
<th>Use of (n)th term. Ans 20 gets 0. Ignore incorrect units. Correct use of (S_n) formula.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(ii)</td>
<td>(S_{26} = 13(600 + 25\times12) = 11700)</td>
<td>M1 A1</td>
<td>[2]</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3) hours (15) minutes.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| (b) | \(ar = 48, ar^2 = 32\) | M1 A1 | [4] | Needs \(ar\) and \(ar^2\) + attempt at \(a\) and \(r\). Correct \(S_n\) formula with \(|r| < 1\) |
|-----|---------------------|-------|-----|-----------------------------------------------------------------|
|     | \(a = 72\). |       |     |                                                                  |
|     | \(S_a = 72 + \frac{72}{3} = 216\). |       |     |                                                                  |

Question 41:
(a) The sum, \(S_n\), of the first \(n\) terms of an arithmetic progression is given by \(S_n = 32n - n^2\). Find the first term and the common difference.

(b) A geometric progression in which all the terms are positive has sum to infinity 20. The sum of the first two terms is 12.8. Find the first term of the progression.

Answer

<table>
<thead>
<tr>
<th>(a)</th>
<th>(S_n = 32n - n^2).</th>
<th>B1</th>
<th>co</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Set (n) to 1, (a) or (S_1 = 31)</td>
<td>M1 A1</td>
<td>[3]</td>
<td>Correct method.</td>
</tr>
<tr>
<td></td>
<td>Set (n) to 2 or other value (S_2 = 60)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(\rightarrow 2)nd term = 29 (\rightarrow d = -2)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(or equates formulae – compares coeffs (n^2, n))</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>([M1\ comparing, A1\ d A1\ a])</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| (b) | \(ar = 48, ar^2 = 32\) | M1 A1 | A1\* | Needs \(ar\) and \(ar^2\) + attempt at \(a\) and \(r\). Correct \(S_n\) formula with \(|r| < 1\) |
|-----|---------------------|-------|-----|-----------------------------------------------------------------|
|     | \(a = 72\). |       |     |                                                                  |
|     | \(S_a = 72 + \frac{72}{3} = 216\). |       |     |                                                                  |
Question 42:
(i) A geometric progression has first term $a$ ($a \neq 0$), common ratio $r$ and sum to infinity $S$. A second geometric progression has first term $a$, common ratio $2r$ and sum to infinity $3S$. Find the value of $r$.

(ii) An arithmetic progression has first term 7. The $n$th term is 84 and the $(3n)$th term is 245. Find the value of $n$.

Answer

(i) $S = \frac{a}{1-r}, \quad 3S = \frac{a}{1-2r}$

$1-r = 3-6r$

$r = \frac{2}{3}$

(ii) $7 + (n-1)d = 84$ and/or $7 + (3n-1)d = 245$

$\left\{ \begin{array}{l} (n-1)d = 77, \quad (3n-1)d = 238, \quad 2nd = 161 \\ n = 23 \quad (d = \frac{77}{22} = 3.5) \end{array} \right.$

Question 43:
The 1st, 2nd and 3rd terms of a geometric progression are the 1st, 9th and 21st terms respectively of an arithmetic progression. The 1st term of each progression is 8 and the common ratio of the geometric progression is $r$, where $r \neq 1$. Find

(i) the value of $r$,

(ii) the 4th term of each progression.

Answer

(i) GP: $8, 8r, 8r^2$

AP: $8, 8+8d, 8+20d$

$8r = 8 + 8d$ and $8r^2 = 8 + 20d$

Eliminates $d$ → $2r^2 - 5r + 3 = 0$

$\rightarrow r = 1.5$ (or 1)

(ii) 4th term of GP: $ar^3 = 8 \times 27/8 = 27$

If $r = 1.5$, $d = 0.5$

4th term of AP: $a + 3d = 9\frac{3}{2}$

Question 44:
Three geometric progressions, $P$, $Q$ and $R$, are such that their sums to infinity are the first three terms respectively of an arithmetic progression.

Progression $P$ is $\quad 2, 1, \frac{1}{2}, \frac{1}{4}, \ldots$

Progression $Q$ is $\quad 3, 1, \frac{1}{3}, \frac{1}{9}, \ldots$

(i) Find the sum to infinity of progression $R$.

(ii) Given that the first term of $R$ is 4, find the sum of the first three terms of $R$. 
Question 45:
An arithmetic progression has first term \(a\) and common difference \(d\). It is given that the sum of the first 200 terms is 4 times the sum of the first 100 terms.

(i) Find \(d\) in terms of \(a\). 

(ii) Find the 100th term in terms of \(a\).

Answer

(i) \[
\frac{200}{2}(2a + 199d) = 4 \times \frac{100}{2}(2a + 99d)
\]
\[
d = 2a \quad \text{cao}
\]

(ii) \[
a + 99d = a + 99 \times 2a
\]
\[
199a \quad \text{cao}
\]

Question 46:
The first term in a progression is 36 and the second term is 32.

(i) Given that the progression is geometric, find the sum to infinity.

(ii) Given instead that the progression is arithmetic, find the number of terms in the progression if the sum of all the terms is 0.

Answer

(i) \[
r = \frac{8}{9}
\]
\[
S_n = (\text{their } a) + (1 - \text{their } r)
\]
\[
S_n = 36 + \frac{1}{9} = 324
\]

(ii) \[
\begin{align*}
\quad & d = -4 \\
\quad & 0 = \frac{n}{2} \left(72 + (n - 1)(-4)\right) \\
\quad & \rightarrow n = 19
\end{align*}
\]

Question 47:
The first term of a progression is \(4x\) and the second term is \(x^2\).

(i) For the case where the progression is arithmetic with a common difference of 12, find the possible values of \(x\) and the corresponding values of the third term.

(ii) For the case where the progression is geometric with a sum to infinity of 8, find the third term.
### Question 48:

(a) The third and fourth terms of a geometric progression are \( \frac{1}{3} \) and \( \frac{2}{9} \) respectively. Find the sum to infinity of the progression.

(b) A circle is divided into 5 sectors in such a way that the angles of the sectors are in arithmetic progression. Given that the angle of the largest sector is 4 times the angle of the smallest sector, find the angle of the largest sector.

### Answer

<table>
<thead>
<tr>
<th>(a)</th>
<th>( a r^2 = \frac{1}{3} ), ( a r^3 = \frac{2}{9} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>→</td>
<td>( r = \frac{2}{3} ) aef</td>
</tr>
<tr>
<td></td>
<td>Substituting ( a = \frac{3}{4} )</td>
</tr>
<tr>
<td>→</td>
<td>( S_n = \frac{3}{4} ) = ( 2 \frac{1}{2} ) aef</td>
</tr>
</tbody>
</table>

Any valid method, seen or implied. Could be answers only.

Both \( a \) and \( r \)

Correct formula with \( |r| < 1 \), cao

<table>
<thead>
<tr>
<th>(b)</th>
<th>( 4a = a + 4d ) → ( 3a = 4d )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( 360 = S_5 = \frac{5}{2}(2a + 4d) ) or 12.5a</td>
</tr>
<tr>
<td>→</td>
<td>( a = 28.8^\circ ) aef</td>
</tr>
<tr>
<td></td>
<td>Largest = ( a + 4d ) or ( 4a = 115.2^\circ ) aef</td>
</tr>
</tbody>
</table>

May be implied in \( 360 = 5/2(a + 4a) \)

Correct \( S_n \) formula or sum of 5 terms

cao, may be implied

(may use degrees or radians)

### Question 49:

A ball is such that when it is dropped from a height of 1 metre it bounces vertically from the ground to a height of 0.96 metres. It continues to bounce on the ground and each time the height the ball reaches is reduced. Two different models, A and B, describe this.

**Model A:** The height reached is reduced by 0.04 metres each time the ball bounces.

**Model B:** The height reached is reduced by 4% each time the ball bounces.
(i) Find the total distance travelled vertically (up and down) by the ball from the 1st time it hits the ground until it hits the ground for the 21st time,

(a) using model A, [3]

(b) using model B. [3]

(ii) Show that, under model B, even if there is no limit to the number of times the ball bounces, the total vertical distance travelled after the first time it hits the ground cannot exceed 48 metres. [2]

Answer

(i) (a) \[ \frac{1.92 + 1.84 + 1.76 + \cdots}{2 \times 1.92 + 19 \times (-0.08)} \approx 0.04 \]

(b) \[ \frac{1.92 + 1.92(0.96) + 1.92(0.96)^2 + \cdots}{1 - 0.96} \approx 26.8 \]

(ii) \[ \frac{1.92}{1 - 0.96} = 48 \text{ or } \frac{0.96}{1 - 0.96} = 24 \text{ & then Double AG} \]

Question 50:

(a) The first term of an arithmetic progression is 2222 and the common difference is 17. Find the value of the first positive term. [3]

(b) The first term of a geometric progression is \( \sqrt{3} \) and the second term is \( 2 \cos \theta \), where \( 0 < \theta < \pi \). Find the set of values of \( \theta \) for which the progression is convergent. [5]

Answer

(a) \[ \frac{2222}{17} (=131 \text{ or } 130.7) \]

(b) \[ r = \frac{2 \cos \theta}{\sqrt{3}} \text{ soi oe} \]

\[ (-1 <) \frac{2 \cos \theta}{\sqrt{3}} < 1 \text{ or } (0 <) \frac{2 \cos \theta}{\sqrt{3}} < 1 \text{ soi oe} \]

\[ \pi/6, 5\pi/6 \text{ soi (but dep. on M1)} \]

\[ \pi/6 < \theta < 5\pi/6 \text{ cao M1A1} \]

Question 51:

A water tank holds 2000 litres when full. A small hole in the base is gradually getting bigger so that each day a greater amount of water is lost.
(i) On the first day after filling, 10 litres of water are lost and this increases by 2 litres each day.

(a) How many litres will be lost on the 30th day after filling? [2]

(b) The tank becomes empty during the nth day after filling. Find the value of n. [3]

(ii) Assume instead that 10 litres of water are lost on the first day and that the amount of water lost increases by 10% on each succeeding day. Find what percentage of the original 2000 litres is left in the tank at the end of the 30th day after filling. [4]

### Answer

![Mathematical expressions and calculations related to the problem.](image)

| Question 52: | The 1st, 3rd and 13th terms of an arithmetic progression are also the 1st, 2nd and 3rd terms respectively of a geometric progression. The first term of each progression is 3. Find the common difference of the arithmetic progression and the common ratio of the geometric progression. [5]

### Answer

![Mathematical expressions and calculations related to the problem.](image)

| Question 53: | The sum of the 1st and 2nd terms of a geometric progression is 50 and the sum of the 2nd and 3rd terms is 30. Find the sum to infinity. [6]

### Answer

![Mathematical expressions and calculations related to the problem.](image)
Question 54:
(a) A cyclist completes a long-distance charity event across Africa. The total distance is 3050 km. He starts the event on May 1st and cycles 200 km on that day. On each subsequent day he reduces the distance cycled by 5 km.

(i) How far will he travel on May 15th? [2]

(ii) On what date will he finish the event? [3]

(b) A geometric progression is such that the third term is 8 times the sixth term, and the sum of the first six terms is $31 \frac{1}{2}$. Find

(i) the first term of the progression, [4]

(ii) the sum to infinity of the progression. [1]

Answer

(a) (i) $200 + (15 - 1)(+/ - 5) = 130$

(ii) $\frac{n}{2} [400 + (n-1)(+/ - 5)] = 3050$

$\rightarrow 5n^2 - 405n + 6100 = 0$

$\rightarrow 20$

M1 A1 [2]

Use of nth term with $a = 200$, $n = 14$ or 15 and $d = +/ - 5$.

(b) (i) $ar^2, ar^5 \rightarrow r = \frac{1}{2}$

$\frac{63}{2} = \frac{a(1-\frac{1}{2^5})}{\frac{1}{2}} \rightarrow a = 16$

M1 A1 [4]

Both terms correct.

Use of $S_6 = 31.5$ with a numeric $r$.

(ii) Sum to infinity $= \frac{16}{\frac{1}{2}} = 32$

B1 [1]

$\sqrt{r}$ for their $a$ and $r$ with $|r| < 1$.

Question 55:
(a) Two convergent geometric progressions, $P$ and $Q$, have the same sum to infinity. The first and second terms of $P$ are 6 and $6r$ respectively. The first and second terms of $Q$ are 12 and $-12r$ respectively. Find the value of the common sum to infinity. [3]

(b) The first term of an arithmetic progression is $\cos \theta$ and the second term is $\cos \theta + \sin^2 \theta$, where $0 \leq \theta \leq \pi$. The sum of the first 13 terms is 52. Find the possible values of $\theta$. [5]

Answer

(a) $\frac{6}{1-r} = \frac{12}{1+r}$

$r = \frac{1}{5}$

$S = 9$

M1 A1 A1 [3]

Question 56:
(a) The first term of a geometric progression in which all the terms are positive is 50. The third term 
is 32. Find the sum to infinity of the progression.

(b) \( \frac{13\pi}{2} \left[ 2 \cos \theta + 12 \sin^2 \theta \right] = 52 \)

\[ 2 \cos \theta + 12(1 - \cos^2 \theta) = 8 \rightarrow 6 \cos^2 \theta - \cos \theta - 2 = 0 \]

\[ \cos \theta = \frac{2}{3} \text{ or } -\frac{1}{2} \text{ soi} \]

\[ \theta = 0.841, \ 2.09 \ \text{Dep on previous A1} \]

(b) The first three terms of an arithmetic progression are \( 2 \sin x, 3 \cos x \) and \( (\sin x + 2 \cos x) \) respectively, where \( x \) is an acute angle.

(i) Show that \( \tan x = \frac{4}{3} \).

(ii) Find the sum of the first twenty terms of the progression.

**Answer**

(a) \( a = 50, \ a_2 = 32 \)

\( \rightarrow r = \frac{4}{5} \) (allow \( -\frac{4}{5} \) for M mark)

\( \rightarrow S_n = 250 \)

(b) (i) \( 2 \sin x, 3 \cos x, (\sin x + 2 \cos x) \).

\( 3e - 2s = (s + 2e) - 3c \)

(or uses \( a, a + d, a + 2d \))

\( \rightarrow 4e = 3s \rightarrow t = \frac{4}{3} \)

SC uses \( t = \frac{4}{3} \) to show

\( u_1 = \frac{8}{5}, u_2 = \frac{9}{5}, u_3 = \frac{10}{5}, \text{BI only} \)

(ii) \( \rightarrow c = \frac{3}{5}, s = \frac{4}{5} \) or calculator \( x = 53.1^\circ \)

\( \rightarrow a = 1.6, \ d = 0.2 \)

\( \rightarrow S_{20} = 70 \)

**Question 57:**

(a) A geometric progression has first term \( 3a \) and common ratio \( r \). A second geometric progression has first term \( a \) and common ratio \( -2r \). The two progressions have the same sum to infinity. Find the value of \( r \).

(b) The first two terms of an arithmetic progression are 15 and 19 respectively. The first two terms of a second arithmetic progression are 420 and 415 respectively. The two progressions have the same sum of the first \( n \) terms. Find the value of \( n \).

**Answer**

(i) \( \frac{3a}{1-r} = \frac{a}{1+2r} \)

\[ 3 + 6r = 1 - r \]

\( r = -\frac{2}{7} \)

\( n = 3 \)

(ii) \( \frac{1}{2}a(2+15+(n-1)d) = \frac{1}{2}a(2+420+(n-1)(-5)) \)

\[ M1A1 \]

\( n = 91 \)
Question 58:
(a) Each year, the value of a certain rare stamp increases by 5% of its value at the beginning of the year. A collector bought the stamp for $10000 at the beginning of 2005. Find its value at the beginning of 2015 correct to the nearest $100.

(b) The sum of the first $n$ terms of an arithmetic progression is $\frac{1}{2}n(3a + 7)$. Find the 1st term and the common difference of the progression.

Answer

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
<th>Marks</th>
<th>Guidance</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>Uses $r = (1.05 \text{ or } 105%)^{10}$</td>
<td>B1</td>
<td>Used to multiply repeatedly or in any GP formula.</td>
</tr>
<tr>
<td>New value $= 10000 \times (1.05)^{10} = 16\ 360$</td>
<td>B1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td><strong>EITHER:</strong> $n = 1 \rightarrow a = 5$</td>
<td>(B1)</td>
<td>Uses $n = 1$ to find $a$</td>
</tr>
<tr>
<td>$n = 2 \rightarrow 13$</td>
<td>B1</td>
<td>Correct $S_n$ for any other value of $n$ (e.g. $n = 2$)</td>
<td></td>
</tr>
<tr>
<td>$a + (a + d) = 13 \rightarrow d = 3$</td>
<td>MI A1</td>
<td>Correct method leading to $d = \cdots$</td>
<td></td>
</tr>
</tbody>
</table>

Q2: $\left(\frac{n}{2}\right)[2a + (n-1)d] = \left(\frac{n}{2}\right)[3a + 7]$ $\left(\frac{n}{2}\right)$ maybe be ignored $\therefore \quad 2a + 2d = 3n + 7 \rightarrow 2a = 3n + 7 \rightarrow d = 3$ $\left(\frac{\text{B1}}{\text{A1}}\right)$ Method mark awarded for equating terms in $n$ from correct $S_n$ formula.

$2a - \{\text{then $3$)}\} = 7, \quad a = 5$ DMI A1)

<table>
<thead>
<tr>
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Question 59:
An arithmetic progression has first term $-12$ and common difference 6. The sum of the first $n$ terms exceeds 3000. Calculate the least possible value of $n$.

Answer

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
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<th>Guidance</th>
</tr>
</thead>
</table>
| (a) | $S_n = \frac{n}{2}[2a + (n-1)d]$ and 20000 | MI | MI correct formula used with $d$ from $16 + d = 24$

A1 | A1 for correct expression linked to 20000. |
| $\rightarrow n^2 + 3n - 5000 < 0, \quad n > 0$ | DM1 | Simplification to a three term quadratic. |
| $\rightarrow (n = 69.2) \rightarrow 70$ terms needed. | A1 | Condense use of 20001 throughout. Correct answer from trial and improvement gets 4/4. |

Question 60:
(a) An arithmetic progression has a first term of 32, a 5th term of 22 and a last term of $-28$. Find the sum of all the terms in the progression.

(b) Each year a school allocates a sum of money for the library. The amount allocated each year increases by 2.5% of the amount allocated the previous year. In 2005 the school allocated $2000. Find the total amount allocated in the years 2005 to 2014 inclusive.
Question 61:
(a) The first two terms of an arithmetic progression are 16 and 24. Find the least number of terms of the progression which must be taken for their sum to exceed 20,000. [4]

(b) A geometric progression has a first term of 6 and a sum to infinity of 18. A new geometric progression is formed by squaring each of the terms of the original progression. Find the sum to infinity of the new progression. [4]

Answer:

(b) \[a = 6, \quad \frac{a}{1-r} = 18 \Rightarrow r = \frac{4}{9}\]

New progression: \[a = 36, \quad r = \frac{4}{9} \text{ oe}\]

New \(S\infty = \frac{36}{1-\frac{4}{9}} = 64.8 \text{ or } \frac{324}{5} \text{ oe}\]

(Total: 4)

Question 62:
The common ratio of a geometric progression is \(r\). The first term of the progression is \((r^2 - 3r + 2)\) and the sum to infinity is \(S\).

(i) Show that \(S = 2 - r\). [2]

(ii) Find the set of possible values that \(S\) can take.

Answer:

(ii) \(1 < S < 3 \text{ or } (1, 3)\) [B2]

\[
1-r \\
(1-r)(2-r) = 2 - r \text{ OE}
\]

(Total: 4)

Question 63:
(a) A geometric progression has a second term of 12 and a sum to infinity of 54. Find the possible values of the first term of the progression. [4]

(b) The \(n\)th term of a progression is \(p + qn\), where \(p\) and \(q\) are constants, and \(S_n\) is the sum of the first \(n\) terms.
Question 64:
The common ratio of a geometric progression is 0.99. Express the sum of the first 100 terms as a percentage of the sum to infinity, giving your answer correct to 2 significant figures. [5]

Answer

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
<th>Marks</th>
<th>Guidance</th>
</tr>
</thead>
<tbody>
<tr>
<td>8(a)</td>
<td>(ar = 12) and (\frac{a}{1-r} = 54)</td>
<td>B1 B1</td>
<td>CAO, OE CAO, OE</td>
</tr>
<tr>
<td></td>
<td>Eliminates (a) or (r) (\rightarrow) (9r^2 - 9r + 2 = 0) or (a^2 - 54a + 648 = 0)</td>
<td>M1</td>
<td>Elimination leading to a 3-term quadratic in (a) or (r)</td>
</tr>
<tr>
<td></td>
<td>(\rightarrow r = \frac{2}{3}) or (\frac{1}{3}) hence to (a \rightarrow a = 18) or 36</td>
<td>A1</td>
<td>Needs both values.</td>
</tr>
</tbody>
</table>

8(b) \(\text{nth term of a progression is} \ p + qn\)
8(b)(i) first term = \(p + q\). Difference = \(q\) or last term = \(p + qn\) | B1 | Need first term and, last term or common difference |
|          | \(S_n = \frac{n}{2} (2(p+q) + (n-1)q)\) or \(\frac{n}{2} (2p + q + qn)\) | MIA1 | Use of \(S_n\) formula with their \(a\) and \(d\). ok unsimplified for A1. |
|          | 3 | |

8(b)(ii) Hence \(2(2p + q + 4q) = 40\) and \(3(2p + q + 6q) = 72\) | DMI | Uses their \(S_n\) formula from (i) |
|          | Solution \(\rightarrow p = 5\) and \(q = 2\) [Could use \(S_n\) with \(a\) and \(d\) \(\rightarrow a = 7\), \(d = 2\) \(\rightarrow p = 5\), \(q = 2\).] | A1 | Note: answers 7, 2 instead of 5, 2 gets M1A0 - must attempt to solve for M1 |

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Question 65:
A company producing salt from sea water changed to a new process. The amount of salt obtained each week increased by 2% of the amount obtained in the preceding week. It is given that in the first week after the change the company obtained 8000 kg of salt.

(i) Find the amount of salt obtained in the 12th week after the change. [3]

(ii) Find the total amount of salt obtained in the first 12 weeks after the change.
Question 66:

(i) The first and second terms of a geometric progression are \( p \) and \( 2p \) respectively, where \( p \) is a positive constant. The sum of the first \( n \) terms is greater than 1000\( p \). Show that \( 2^n > 1001 \). [2]

(ii) In another case, \( p \) and \( 2p \) are the first and second terms respectively of an arithmetic progression. The \( n \)th term is 336 and the sum of the first \( n \) terms is 7224. Write down two equations in \( n \) and \( p \) and hence find the values of \( n \) and \( p \). [5]

Answer

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
<th>Marks</th>
<th>Guidance</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) ( S_n = \frac{p(2^n - 1)}{2 - 1} )</td>
<td>M1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( p(2^n - 1) &gt; 1000p \Rightarrow 2^n &gt; 1001 )</td>
<td>A1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(ii) ( p + (n - 1)p = 336 )</td>
<td>B1</td>
<td>Expect ( np = 336 )</td>
<td></td>
</tr>
<tr>
<td>( \frac{n}{2}[2p + (n - 1)p] = 7224 )</td>
<td>B1</td>
<td>Expect ( \frac{n}{2}(p + np) = 7224 )</td>
<td></td>
</tr>
<tr>
<td>Eliminate ( n ) or ( p ) to an equation in one variable</td>
<td>M1</td>
<td>Expect e.g. ( 168(1 + n) = 7224 ) or ( 1 + 336/p = 43 ) etc</td>
<td></td>
</tr>
<tr>
<td>( n = 42, p = 8 )</td>
<td>A1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td></td>
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Question 67:

(a) The third and fourth terms of a geometric progression are 48 and 32 respectively. Find the sum to infinity of the progression. [3]

(b) Two schemes are proposed for increasing the amount of household waste that is recycled each week.

Scheme A is to increase the amount of waste recycled each month by 0.16 tonnes.

Scheme B is to increase the amount of waste recycled each month by 6% of the amount recycled in the previous month.

The proposal is to operate the scheme for a period of 24 months. The amount recycled in the first month is 2.5 tonnes.

For each scheme, find the total amount of waste that would be recycled over the 24-month period.
Answer:

<table>
<thead>
<tr>
<th></th>
<th>Answer</th>
<th>Marks</th>
<th>Guidance</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>$\sigma^2 = 48, ; \sigma^4 = 32, ; r = \frac{1}{3} ; \text{or} ; \sigma = 108$</td>
<td>M1</td>
<td>Solution of the 2 eqns to give $r$ (or $\sigma$). A1 (both)</td>
</tr>
<tr>
<td></td>
<td>$r = \frac{1}{3} ; \text{and} ; \sigma = 108$</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$s_2 = \frac{108}{1} = 324$</td>
<td>A1</td>
<td>FT Needs correct formula and $r$ between $-1$ and $1$.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$3$</td>
</tr>
<tr>
<td>(b)</td>
<td>Scheme A $\sigma = 2.50, ; d = 0.16$</td>
<td>M1</td>
<td>Correct use of either AP $s_n$ formula.</td>
</tr>
<tr>
<td></td>
<td>$s_n = 12(5 + 23 \times 0.16)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$s_n = 104 ; \text{tonnes.}$</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Scheme B $\sigma = 2.50, ; r = 1.08$</td>
<td>B1</td>
<td>Correct value of $r$ used in GP.</td>
</tr>
<tr>
<td></td>
<td>$\frac{2.5(1.06^{2n} - 1)}{1.06 - 1}$</td>
<td>M1</td>
<td>Correct use of either $s_n$ formula.</td>
</tr>
<tr>
<td></td>
<td>$s_n = 127 ; \text{tonnes.}$</td>
<td>A1</td>
<td></td>
</tr>
</tbody>
</table>

Question 68:

(a) In an arithmetic progression, the sum of the first ten terms is equal to the sum of the next five terms. The first term is $a$.

(i) Show that the common difference of the progression is $\frac{1}{3}a$.        [4]

(ii) Given that the tenth term is 36 more than the fourth term, find the value of $a$. [2]

(b) The sum to infinity of a geometric progression is 9 times the sum of the first four terms. Given that the first term is 12, find the value of the fifth term. [4]

Answer:

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
<th>Marks</th>
<th>Guidance</th>
</tr>
</thead>
<tbody>
<tr>
<td>10(a)(i)</td>
<td>$s_{10} = s_{10} = s_{10} = s_{10}$</td>
<td>M1</td>
<td>Either statement seen or implied.</td>
</tr>
<tr>
<td></td>
<td>$5(2a + 9d) ; \text{ce}$</td>
<td>B1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$7.5(2a + 14d) - 5(2a + 9d) ; \text{or} ; \frac{5}{2} (a + 10d) + (a+14d) ; \text{ce}$</td>
<td>A1</td>
<td>Correct answer from convincing working</td>
</tr>
<tr>
<td></td>
<td>$d = \frac{a}{3} ; \text{AG}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10(a)(ii)</td>
<td>$(a + 9d) = 36 + (a + 3d)$</td>
<td>M1</td>
<td>Correct use of $a+(n-1)d$ twice and addition of $\pm 36$</td>
</tr>
<tr>
<td></td>
<td>$a = 18$</td>
<td>A1</td>
<td></td>
</tr>
</tbody>
</table>

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Question 69:

(i) The first, second and third terms of a geometric progression are \(x, x - 3\) and \(x - 5\) respectively.

(ii) Find the value of \(x\).

(iii) Find the fourth term of the progression.

(iv) Find the sum to infinity of the progression.

**Answer:**

\[
\begin{align*}
\text{(b)(i)} & \quad \frac{x-3}{x} = \frac{x-5}{x-3} \quad \text{(or use of } a, ar \text{ and } ar^2) \quad \text{M1} \quad \text{Any valid method to obtain an equation in one variable.} \\
& \quad \therefore a = \frac{x}{3} \quad \text{A1} \\
& \quad \therefore 2 \\
\text{(b)(ii)} & \quad r = \frac{x-3}{x} \quad \text{A1 OE, AWRT} \\
& \quad \text{(or use of } a, ar \text{ and } ar^2) \quad \text{M1} \quad \text{Any valid method to find } r \text{ and the fourth term with their } a \text{ & } r. \\
& \quad \text{Fourth term} = 9 \times \left(\frac{3}{3}\right)^4 \\
& \quad \text{4% or 2.67} \quad \text{A1} \quad \text{AWRT} \\
\text{(b)(iii)} & \quad S_{\infty} = \frac{a}{1-r} = \frac{9}{1-\frac{3}{2}} \quad \text{M1} \quad \text{Correct formula and using their } r \text{ and } a, \text{ with } |r| < 1, \text{ to obtain a numerical answer.} \\
& \quad \text{27 or 27.0} \quad \text{A1} \quad \text{AWRT}
\end{align*}
\]

Question 70:

The first, second and third terms of a geometric progression are \(3k, 5k - 6\) and \(6k - 4\), respectively.

(i) Show that \(k\) satisfies the equation \(7k^2 - 48k + 36 = 0\).

(ii) Find, showing all necessary working, the exact values of the common ratio corresponding to each of the possible values of \(k\).

(iii) One of these ratios gives a progression which is convergent. Find the sum to infinity.

**Answer:**

\[
\begin{align*}
\text{(i)} & \quad \frac{5k-6}{3k} = \frac{6k-4}{5k-6} \quad \text{M1} \quad \text{OR any valid relationship} \\
& \quad \therefore 25k^2 - 60k + 36 = 18k^2 - 12k \\
& \quad \therefore 7k^2 - 48k + 36 \\
& \quad \therefore A1 \quad \text{AG} \\
\text{(ii)} & \quad k = \frac{6}{7} \\
& \quad \text{B1B1} \quad \text{Allow 0.857(1) for } \frac{6}{7} \\
& \quad \text{When } k = \frac{6}{7}, r = -\frac{2}{3} \\
& \quad \text{B1} \quad \text{Must be exact} \\
& \quad \text{When } k = 6, r = \frac{4}{3} \\
\text{(iii)} & \quad \text{Use of } S_{\infty} = \frac{a}{1-r} \quad \text{with } r = \text{their } \frac{2}{3} \text{ and } a = 3 \times \text{their } \frac{6}{7} \\
& \quad \text{M1} \quad \text{Provided } 0 < |\text{their } -2/3| < 1 \\
& \quad \frac{18}{7} \times \left(1 + \frac{2}{3}\right) = \frac{54}{35} \text{ or 1.54} \quad \text{A1} \quad \text{FT if 0.857(1) has been used in part (ii)}
\end{align*}
\]
**Question 71:**

(a) An arithmetic progression has a first term of 5 and a common difference of -3.

Find the number of terms such that the sum to \( n \) terms is first less than -200. \( \text{[4]} \)

(b) A geometric progression is such that its 3rd term is equal to \( \frac{81}{64} \) and its 5th term is equal to \( \frac{729}{1024} \).

(i) Find the first term of this progression and the positive common ratio of this progression. \( \text{[5]} \)

(ii) Hence find the sum to infinity of this progression. \( \text{[1]} \)

**Answer:**

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
<th>Marks</th>
<th>Partial Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>10(a)</td>
<td>(-200 &gt; \frac{n}{2}(-10 + (n-1)(-3))) leading to (3(r^2 - 13r - 400) &gt; 0) (n = 13.9)... so 14th term needed</td>
<td>4</td>
<td>MI for attempt to use sum to ( n ) terms, allow use of ( n ) or ( r ) or ( a ),  1A for correct quadratic expression, DMI for attempt to solve, A1 for correct conclusion</td>
</tr>
</tbody>
</table>
| 10(b)(i) | \(ar^2 = \frac{81}{64}\)  
           | \(ar^4 = \frac{729}{1024}\)  
           | \(r = \frac{9}{16}\)  
           | \(r = \frac{3}{4}\)  
           | \(a = 9\)  | 5 | B1 for 3rd term, B1 for 5th term, MI for attempt to solve their equations to obtain either \( r \) or \( a \), A1 for \( r \), A1 for \( a \) |
| 10(b)(ii) | \(S_n = 9\) | B1 | FT on their \( a \) and \( r \) provided \(|r| < 1\) |
Inequalities Past Papers 1972-2019

1. Find the solution set of the inequality \( \frac{4x - 5}{1-x} > 1 \).  
   \( (J72/P2/1) \)

2. Find the solution set of the inequality \( \frac{10}{x} < 19 - 6x \).  
   \( (N72/P1/2) \)

3. Find the solution set of the inequality \( \frac{x-4}{x-2} > 3 \).  
   \( (J73/P2/1) \)

4. In answering either part of this question, you may, if you wish, make use of rough sketch graphs.
   (a) Given that the inequalities \( x + y > 1, 3y > 2x - 1, 3x > 2y \) are simultaneously satisfied, find the range of values to which \( x \) is restricted and the range of values to which \( y \) is restricted.
   (b) Find the solution set of the inequality \( x + 1 < \frac{3}{2} \).  
      \( (N73/P1/2) \)

5. Find the solution set of the inequality \( \frac{12}{x-3} < x + 1 \).  
   \( (J74/P1/2) \)

6. (a) Calculate the area of the region of the \( x - y \) plane defined by the simultaneous inequalities \( y \leq 2x, x + y \leq 6, x \leq 5y \).
   (b) Find the solution set of the inequality \( \frac{x-1}{x+1} > \frac{x}{6} \).  
      \( (N74/P2/2) \)

7. Prove that the simultaneous inequalities \( y < x, 2y + x > 0, x + y < 12 \) together imply \( 0 < x < 24 \), and find the set of values to which \( y \) is restricted.  
   \( (J75/P2/2) \)

8. Find the solution set of the inequalities
   (a) \( \frac{3}{2} < \frac{2x - 3}{x - 5} \),  
   (b) \( 3x(x - 5) < 2(2x - 3) \).  
      \( (N75/P2/1) \)

9. Use a graphical method to find all pairs of positive integers \( (x, y) \) satisfying the following three inequalities simultaneously: \( 4x - y > 2, 2x + y < 12, y > x \).  
   \( (J76/P2/2) \)

10. Draw a diagram illustrating the region \( S \) of the \( x - y \) plane which is defined by the simultaneous inequalities \( x + y \geq 7, 2x + y \leq 13, 2x + 3y \leq 19 \), and give the coordinates of the vertices of \( S \). Prove that, if the line \( y = kx \) intersects \( S \), then \( \frac{1}{6} \leq k \leq \frac{7}{2} \). The point \( P \) lies on \( y = kx \) and is in the region \( S \). Prove that, when \( \frac{1}{6} \leq k \leq \frac{7}{2} \), the maximum value for the \( y \)-coordinate of \( P \) is \( 13k/(2 + k) \), and find the corresponding expression when \( \frac{3}{5} \leq k \leq \frac{12}{5} \).  
    \( (J77/P1/2) \)

11. Find the solution set of the inequality \( \frac{2}{x-1} < \frac{1}{x+1} \) where \( x \in R, x \neq 1, -1 \).  
    \( (J78/P2/3) \)

12. The set \( S \) is \( \{(x, y) ; y + 2x \geq 5 \text{ and } x^2 + y^2 \leq 10, (x, y) \in R \times R \} \). Give a sketch of the \( x - y \) plane to show the region in which the points representing the members of \( S \) must lie. If \( (x, kx) \in S \) find the set of possible values of \( k \).  
    \( (N78/P1/1) \)

13. (a) Show that the region of the \( x - y \) plane within which the following four simultaneous inequalities are satisfied is, in fact, defined by only three of the inequalities, and state the redundant inequality. \( x < 6, y < x, 3x - y > 3, x + 2y > 6 \).
   (b) Find the solution set of the inequality \( \frac{2}{x+1} > x, x \in R, x \neq -1 \).  
    \( (J79/P2/2) \)

14. Solve the inequality \( \frac{12}{x-2} > 3x - 2, (x \in R, x \neq 2) \).  
    \( (N79/P1/1) \)

15. (a) Find the solution set for \( x \) given that the following three relations for \( x, y \), where \( x, y \in R \), are simultaneously true: \( y < x + 1, y + 6x < 20, x = 5y - 7 \).
   (b) Find the solution set of the inequality \( \frac{12}{x-3} < x + 1, (x \in R, x \neq 3) \).  
    \( (J80/P2/2) \)
16. (a) Functions \( f \) and \( g \) are defined by \( f : x \to x(x - 1) \), \( g : x \to (x - 1)(3x - 5) \), where \( x \in R \) in each case.

(ii) Find the solution set, \( S \), of the inequality \( f(x) \geq g(x) \).

(iii) Sketch the graph of \( y = f(x) - g(x) \) for \( x \in S \), and state the greatest and least values of \( f(x) - g(x) \) for \( x \in S \).

(b) Illustrate by means of a sketch the subset of the \( x \)-\( y \) plane given by \( \{(x, y) : |x| < |y|\} \). (J81/P1/2)

17. (a) The set \( S \) of ordered pairs of real numbers is given by \( S = \{(x, y) : y \geq 3x, y + 2x \leq 35, y \leq x^4\} \). Draw a sketch showing, by shading, the region of the \( x \)-\( y \) plane containing all the points \((x, y)\) in \( S \). Given that \((x, y) \in S\), find the maximum value of \((4x - y)\) and the minimum value of \((x + y)\).

(b) Find the solution set of the inequality \( \frac{6}{x - 1} > x \), \((x \in R, x \neq 1)\). (N81/P1/2)

18. Solve the inequality \( \frac{2x}{x + 1} > x \), \((x \in R, x \neq -1)\). (J82/P2/1)

19. Find the set of values of \( k \) for which, for all real values of \( x \), \( 3x^2 + 3x + k > 0 \) and \( 3x^2 + kx + 3 > 0 \). (N82/P2/1)

20. Find the solution set of the inequality \( \frac{1}{2 - x} < \frac{1}{x - 3} \). (N82/P2/1)

21. (a) Find the solution set of the inequality \( \frac{x - 1}{x + 1} + 1 > 0 \).

(b) Given that \( x + 2y \geq 3 \) and \( y - 3x \geq 5 \), show that \( y - x \geq 3 \). (J83/P2/3)

22. The set, \( S \), of ordered pairs, \((x, y)\), of real numbers is defined by \( S = \{(x, y) : y - 2x \leq 0, 2y - x \geq 0, 2y + x - 20 \leq 0\} \). Illustrate the region in the \( x \)-\( y \) plane determined by the set \( S \). For \((x, y) \in S\) find

(a) the greatest value of \( x + 4y \),

(b) the greatest value of \( x^2 + y^2 \),

(c) the set of values of \( y^2 - 6y \). (J83/P1/2)

23. (a) Solve the inequality \( \frac{3}{1-x} < 5 - 4x \), \((x \in R, x \neq 1)\).

(b) Draw a sketch to illustrate the region \( R \) of the \( x \)-\( y \) plane defined by the simultaneous inequalities \( 3x - 7y \geq 1, 2x + y \leq 12 \). Show that the line \( y = mx + (2 - 5m) \) passes through the vertex of \( R \) for all values of \( m \). Deduce the set of values of \( m \) for which the inequality \( y \leq mx + (2 - 5m) \) is true for all \((x, y) \in R\). (J84/P2/3)

24. (a) Illustrate the solution set of the simultaneous inequalities \( 9 \leq y + 3x \leq 18, 0 \leq 2y - 3x \leq 18 \) by means of a diagram, and write down the sets of values to which \( x \) and \( y \) are separately restricted.

(b) Find the solution set of the inequality \( \frac{2}{x - 3} > \frac{3}{x - 2} \), where \( x \in R, x \neq 2, x \neq 3 \). (J85/P2/2)

25. Find the solution set of the inequalities:

(a) \(|x - 2| < 2x\)

(b) \(\frac{6}{|x| + 1} < |x|\)

(c) \(x^2 - 6x + 7 > 0\)

26. Find the range(s) of the values of \( x \) for which the following inequalities hold:

(a) \(|3x + 5| < 4\)

(b) \(|\frac{2x + 1}{x - 3}| > 1\)

(c) \(|x^2 + 1| < |x^2 - 9|\)

(d) \(|3 - 2x| \leq |x + 4|\)
27. (a) Find the solution set of the inequality \( x + \frac{1}{x} > \frac{5}{2} \).
(b) Solve the simultaneous equations \( x + y = 6 \), \( x^2 + y^2 = 26 \). Hence, with the aid of a diagram, or otherwise, determine all the pairs of integers \((x, y)\) for which the inequalities \( x + y \geq 6 \), \( x^2 + y^2 \leq 26 \) are simultaneously true.

(N85/P1/2)

28. Solve the following inequalities:
   (a) \( \frac{x+1}{x-1} < 4 \), \( \frac{|x|+1}{|x|-1} < 4 \), \( \frac{|x+1|}{|x|-1} < 4 \).
   (b) \( \frac{x+1}{x-1} < 4 \).
   (c) \( \frac{x+1}{x-1} < 4 \).
   (J87/P1/18)

29. Sketch the graph of \( y = |x+2| \) and hence, or otherwise, solve the inequality \( |x+2| > 2x+1, x \in R \).
   (N87/P1/4)

30. Solve the inequality \( x - x^3 > 0 \).
   (J88/P2/5)

31. (a) Sketch, on the same diagram, the graphs of \( y = \frac{1}{x} \) and \( y = x - \frac{3}{2} \). Find the solution set of the inequality \( x - \frac{3}{2} > \frac{1}{x} \).
   (b) Sketch, on separate diagrams, the graphs of \( y = |x|, \ y = |x-3|, \ y = |x-3| + |x+3| \). Find the solution set of the equation \( |x-3| + |x+3| = 6 \).
   (J89/P1/14)

32. Solve the inequality \( (x-1)(x-2) > 0 \).
   (N89/P1/6)

33. Solve, for \( x \in R \), each of the following inequalities:
   (a) \( \frac{x}{x-2} < 5 \), \( \frac{x}{x-2} < 5 \), \( \frac{|x|}{x-2} < 5 \).
   (b) \( x(x-2) < 5 \), \( x(x-2) < 5 \), \( x |x| < 4 |x-3| \).
   (c) \( |x| < 4 |x-3| \).
   (J90/P1/15)

34. Solve the equation \( 4 |x| = |x-1| \). On the same diagram, sketch the graphs of \( y = 4 |x| \) and \( y = |x-1| \) and hence, or otherwise, solve the inequality \( 4 |x| > |x-1| \).
   (N90/P1/4)

35. Solve the inequality \( x^2 < 6x - x^2 \).
   (J91/P1/3)

36. Solve the inequality \( \frac{x+5}{x-2} < 3 \).
   (J92/P1/4)

37. Sketch, on a single diagram, the graphs of \( x + 2y = 6 \) and \( y = |x+2| \). Hence, or otherwise, solve the inequality \( |x + 2| < \frac{1}{2}(6-x) \).
   (N92/P1/4)

38. Express \( 3x^2 - 12x - 4 \) in the form \( A(x + B)^2 + C \), giving the values of \( A, B \) and \( C \). Hence or otherwise solve the inequality \( 3x^2 - 12x - 4 > 0 \). Give your answer in a form involving one or more of the intervals \( x < a, x > b, a < x < b \), where the value of \( a \) and \( b \) contain surds. Hence or otherwise solve exactly the inequalities
   (a) \( 3y^2 + 12y - 4 > 0 \),
   (b) \( 3e^{2x} - 12e^x - 4 > 0 \).
   (N93/P1/12)

Question 39:
Solve the inequality \( |x + 2| < |5 - 2x| \).

Answer:

\textit{Either}: State or imply non-modular inequality \((x + 2)^2 < (5 - 2x)^2\), or corresponding equation \( B1 \).
Expand and make reasonable solution attempt at 2- or 3-term quadratic, or equivalent \( M1 \).
Obtain critical values 1 and 7 \( A1 \).
State correct answer \( x < 1, x > 7 \) \( A1 \).

\textit{Or}: State one correct equation for a critical value e.g. \( x + 2 = 5 - 2x \) \( M1 \).
State two relevant equations separately e.g. \( x + 2 = 5 - 2x \) and \( x + 2 = -(5 - 2x) \) \( A1 \).
Obtain critical values 1 and 7 \( A1 \).
State correct answer \( x < 1, x > 7 \) \( A1 \).
Question 40:
Solve the inequality \(|9 - 2x| < 1|.

Answers:

**EITHER:** State or imply non-modular inequality \((9 - 2x)^2 < 1\), or a correct pair of linear inequalities, combined or separate, e.g. \(-1 < 9 - 2x < 1\).
- Obtain both critical values 4 and 5
- State correct answer \(4 < x < 5\); accept \(x > 4, x < 5\)
- B1

**OR:** State a correct equation or pair of equations for both critical values e.g. \(9 - 2x = 1\) and \(9 - 2x = -1\), or \(9 - 2x = \pm 1\)
- Obtain critical values 4 and 5
- State correct answer \(4 < x < 5\); accept \(x > 4, x < 5\)
- B1

**OR:** State one critical value (probably \(x = 4\)) from a graphical method or by inspection or by solving a linear inequality or equation
- State the other critical value correctly
- State correct answer \(4 < x < 5\); accept \(x > 4, x < 5\)
- [Use of ≤, throughout, or at the end, scores a maximum of B2.]

[3] 9709/2/M/J/02

Question 41:
Solve the inequality \(|x - 2| < 3 - 2x|.

Answers:

**EITHER** State or imply non-modular inequality \((x-2)^2 < (3-2x)^2\), or corresponding equation
- Expand and make a reasonable solution attempt at a 2- or 3-term quadratic, or equivalent
- Obtain critical value \(x = 1\)
- State answer \(x < 1\) only
- B1 A1 A1

**OR** State the relevant linear equation for a critical value, i.e. \(2 - x = 3 - 2x\), or equivalent
- Obtain critical value \(x = 1\)
- State answer \(x < 1\)
- State or imply by omission that no other answer exists
- B1

**OR** Obtain the critical value \(x = 1\) from a graphical method, or by inspection, or by solving a linear inequality
- State answer \(x < 1\)
- State or imply by omission that no other answer exists
- B1

[4] 9709/3/O/N/02

Question 42:
Solve the inequality \(|x - 4| > |x + 1|.

Answers:

**EITHER** State or imply non-modular inequality \((x - 4)^2 > (x + 1)^2\), or corresponding equation
- Expand and solve a linear inequality, or equivalent
- Obtain critical value \(1\frac{1}{2}\)
- State correct answer \(x < 1\frac{1}{2}\) (allow ≤)
- B1

[4] 9709/3/M/J/03
Question 43:
Find the set of values of $x$ satisfying the inequality $|8 - 3x| < 2$. [3]

Answers:
1. **EITHER:** State or imply non-modular inequality e.g. $-2 < 8 - 3x < 2$, or $(8 - 3x)^2 < 2^2$, or corresponding equation or pair of equations M1
   - Obtain critical values $2$ and $3 \frac{1}{3}$ B1
   - State correct answer $2 < x < 3 \frac{1}{3}$ A1

2. **OR:** State one critical value (probably $x = 2$), from a graphical method or by inspection or by solving a linear equation or equation B1
   - State the other critical value correctly B1
   - State correct answer $2 < x < 3 \frac{1}{3}$ B1

[3] 9709/02/O/N/03

Question 44:
Solve the inequality $|2^x - 8| < 5$. [4]

Answers:
**EITHER:** State or imply non-modular inequality $-5 < 2^x - 8 < 5$, or $(2^x - 8)^2 < 5^2$ or corresponding pair of linear equations or quadratic equation B1
   - Use correct method for solving an equation of the form $2^x = a$ M1
   - Obtain critical values $1.58$ and $3.70$, or exact equivalents A1
   - State correct answer $1.58 < x < 3.70$ A1

**OR:** Use correct method for solving an equation of the form $2^x = a$ M1
   - Obtain one critical value (probably $3.70$), or exact equivalent A1
   - Obtain the other critical value, or exact equivalent A1
   - State correct answer $1.58 < x < 3.70$ A1

[Allow $1.59$ and $3.7$. Condone ≤ for <. Allow final answers given separately. Exact equivalents must be in terms of ln or logarithms to base 10.]
[SR: Solutions given as logarithms to base 2 can only earn M1 and B1 of the first scheme.]

9709/03/O/N/03

Question 45:
Solve the inequality $|2x + 1| < |x|$. [4]

Answers:
**EITHER:** State or imply non-modular inequality $(2x + 1)^2 < x^2$ or corresponding quadratic equation or pair of linear equations $(2x + 1)^2 = \pm x$ B1
   - Expand and make a reasonable solution attempt at a 3-term quadratic, or solve two linear equations M1
   - Obtain critical values $x = -1$ and $x = -\frac{1}{3}$ only A1
   - State answer $-1 < x < -\frac{1}{3}$ A1
Compiled By: Naqash Sachwani

Question 46:
Solve the inequality \(|x + 1| > |x|\).

Answers:

\textit{EITHER:} State or imply non-modular inequality \((x + 1)^2 > x^2\), or corresponding quadratic equation or linear equation \(x + 1 = -x\)

\[\text{Obtain critical value } -\frac{1}{2}\]

\[\text{State answer } x > -\frac{1}{2}\]

\textit{OR:} Obtain critical value \(-\frac{1}{2}\) by solving a linear inequality or by graphical method or inspection

\[\text{State answer } x > -\frac{1}{2}\]

[For \(2x + 1 > 0, x > -\frac{1}{2}\), or similar reasonable method]

9709/02/O/N/04

Question 47:
Solve the inequality \(|x| > |3x - 2|\).

Answers:

\textit{EITHER} State or imply non-modular inequality \(x^2 > (3x - 2)^2\), or corresponding equation

Expand and make reasonable solution attempt at 2- or 3-term quadratic, or equivalent

\[\text{Obtain critical values } \frac{1}{2} \text{ and } 1\]

\[\text{State correct answer } \frac{1}{2} < x < 1\]

\textit{OR} State one correct linear equation for a critical value

\[\text{State two equations separately}\]

\[\text{Obtain critical values } \frac{1}{2} \text{ and } 1\]

\[\text{State correct answer } \frac{1}{2} < x < 1\]

\textit{OR} State one critical value from a graphical method or inspection or by solving a linear inequality

\[\text{State the other critical value correctly}\]

\[\text{State correct answer } \frac{1}{2} < x < 1\]

9709/02/M/J/05

Question 48:
Given that \(a\) is a positive constant, solve the inequality

\[|x - 3a| > |x - a|\] \hfill [4]

Answer:

\textit{EITHER:} State or imply non-modular inequality \((x-3a)^2 > (x-a)^2\), or corresponding equation

Expand and solve the inequality, or equivalent

\[\text{Obtain critical value } 2a\]

\[\text{State correct answer } x < 2a \text{ only}\]

\textit{OR:} State a correct linear equation for the critical value, e.g. \(x-3a = -(x-a)\), or corresponding inequality

\[\text{Solve the linear equation for } x, \text{ or equivalent}\]

\[\text{Obtain critical value } 2a\]

\[\text{State correct answer } x < 2a \text{ only}\]
Question 49:
Solve the inequality $2x > |x - 1|$.  

Answer:

EITHER: State or imply non-modular inequality $(2x)^2 > (x - 1)^2$, or corresponding equation  
Expand and make a reasonable solution attempt at a 2- or 3-term quadratic  
Obtain critical value $x = \frac{1}{3}$  
State answer $x > \frac{1}{3}$ only  

OR: State the relevant critical linear equation, i.e. $2x = 1 - x$  
Obtain critical value $x = \frac{1}{3}$  
State answer $x > \frac{1}{3}$  
State or imply by omission that no other answer exists  

OR: Obtain the critical value $x = \frac{1}{3}$ from a graphical method, or by inspection, or by solving a linear inequality  
State answer $x > \frac{1}{3}$  
State or imply by omission that no other answer exists  

Question 50:
Solve the inequality $|2x - 1| > |x|$.  

Answers:

EITHER: State or imply non-modular inequality $(2x - 1)^2 > x^2$ or corresponding quadratic equation  
or pair of linear equations $2x - 1 = \pm x$  
Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations  
Obtain critical values $x = 1$ and $x = \frac{1}{3}$  
State answer $x < \frac{1}{3}, x > 1$  

OR: Obtain critical value $x = 1$ from a graphical method, or by inspection, or by solving a linear inequality or linear equation  
Obtain the critical value $x = \frac{1}{3}$ similarly  
State answer $x < \frac{1}{3}, x > 1$

Question 51:
(i) Express $4^x$ in terms of $y$, where $y = 2^x$.  

(ii) Hence find the values of $x$ that satisfy the equation  

$$3(4^x) - 10(2^x) + 3 = 0,$$

giving your answers correct to 2 decimal places.  

Answers:

(i) State or imply that $4^x = y^2$ (or $2^{2x}$)  

(ii) Carry out recognizable solution method for a quadratic equation in $y$  

Obtain $y = 3$ and $y = \frac{1}{3}$ from $3y^2 - 10y + 3 = 0$  

Use logarithmic method to solve an equation of the form $2^x = k$, where $k > 0$  

State answer 1.58  

State answer $-1.58$  

(A1 √ if ± 1.59)
Question 52:
Solve the inequality $|x - 3| > |x + 2|$.

Answers:
EITHER
State or imply non-modular inequality $(x - 3)^2 > (x + 2)^2$, or corresponding equation
M1
Expand and solve a linear inequality, or equivalent
M1
Obtain critical value $\frac{1}{2}$
A1
State correct answer $x < \frac{1}{2}$ (allow $x < \frac{1}{2}$)
A1

OR
State a correct linear equation for the critical value, e.g. $3 - x = x + 2$,
or corresponding correct inequality, e.g. $-(x - 3) > (x + 2)$
M1
Solve the linear equation, or inequality
M1
Obtain critical value $\frac{1}{2}$
A1
State correct answer $x < \frac{1}{2}$
A1

OR
Make recognisable sketches of both $y = |x - 3|$ and $y = |x + 2|$ on a single diagram
B1
Obtain a critical value from the intersection of the graphs
M1
Obtain critical value $\frac{1}{2}$
A1
State final answer $x < \frac{1}{2}$
A1

9709/02/M/J/07

Question 53:
(i) Solve the inequality $|y - 5| < 1$.

(ii) Hence solve the inequality $|3y - 5| < 1$, giving 3 significant figures in your answer.

Answers:
(i) Obtain critical values 4 and 6
B1
State answer $4 < y < 6$
B1

(ii) Use correct method for solving an equation of the form $3^x = a$, where $a > 0$
M1
Obtain one critical value, i.e. either 1.26 or 1.63
A1
State answer $1.26 < x < 1.63$
A1

9709/02/O/N/07

Question 54:
Solve the inequality $|x - 2| > 3|2x + 1|$.

Answers:
EITHER
State or imply non-modular inequality $(x - 2)^2 > (3(2x + 1))^2$, or corresponding quadratic equation, or pair of linear equations
B1
$(x - 2) = \pm 3(2x + 1)$
M1
Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations
M1
Obtain critical values $x = -1$ and $x = -\frac{1}{7}$
A1
State answer $-1 < x < -\frac{1}{7}$
A1
Question 55:
Solve the inequality $|x - 3| > |2x|$. 

Answers:

EITHER: State or imply non-modular inequality $(x - 3)^2 > (2x)^2$ or corresponding quadratic equation or pair of linear equations $(x - 3) = \pm 2x$ or $(x - 3) = \mp 2x$ or pair of linear equations $3x + 2 = \pm x$ M1 
Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations M1 
Obtain critical values $x = 1$ and $x = -3$ A1 
State answer $-3 < x < 1$ A1

OR: Obtain critical value $x = -3$ from a graphical method, or by inspection, or by solving a linear inequality or linear equation B1 
Obtain the critical value $x = 1$ similarly B2 
State answer $-3 < x < 1$ B1

Question 56:
Solve the inequality $|3x + 2| < |x|$. 

Answers:

EITHER: State or imply non-modular inequality $(3x + 2)^2 < x^2$, or corresponding quadratic equation, or pair of linear equations $3x + 2 = \pm x$ M1 
Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations M1 
Obtain critical values $x = -1$ and $x = -\frac{1}{2}$ A1 
State answer $-1 < x < -\frac{1}{2}$ A1

OR: Obtain the critical value $x = -1$ from a graphical method or by inspection, or by solving a linear equation or inequality B1 
Obtain the critical value $x = -\frac{1}{2}$ similarly B2 
State answer $-1 < x < -\frac{1}{2}$ B1

Question 57:
Solve the inequality $2 - 3x < |x - 3|$. 

Answers:

EITHER: State or imply non-modular inequality $(2 - 3x)^2 < (x - 3)^2$, or corresponding equation, and make a reasonable solution attempt at a 3-term quadratic M1 
Obtain critical value $x = -\frac{1}{2}$ A1 
Obtain $x > -\frac{1}{2}$ A1 
Fully justify $x > -\frac{1}{2}$ as only answer A1

OR1: State the relevant critical linear equation, i.e. $2 - 3x = 3 - x$ B1 
Obtain critical value $x = -\frac{1}{2}$ B1 
Obtain $x > -\frac{1}{2}$ B1 
Fully justify $x > -\frac{1}{2}$ as only answer B1

OR2: Obtain the critical value $x = -\frac{1}{2}$ by inspection, or by solving a linear inequality B2 
Obtain $x > -\frac{1}{2}$ B1 
Fully justify $x > -\frac{1}{2}$ as only answer B1
Question 58:
Solve the equation $3^{x+2} = 3^x + 3^2$, giving your answer correct to 3 significant figures.  [4]

Answers:

**EITHER:** Use laws of indices correctly and solve a linear equation for $3^x$, or for $3^{-x}$  M1

Obtain $3^x$, or $3^{-x}$ in any correct form, e.g. $3^x = \frac{3^2}{(3^2 - 1)}$  A1

Use correct method for solving $3^x = a$ for $x$, where $a > 0$  M1

Obtain answer $x = 0.107$  A1

**OR:** State an appropriate iterative formula, e.g. $x_{n+1} = \frac{\ln(3^x + 9)}{\ln 3} - 2$  B1

Use the formula correctly at least once  M1

Obtain answer $x = 0.107$  A1

Show that the equation has no other root but 0.107  A1

[For the solution 0.107 with no relevant working, award B1 and a further B1 if 0.107 is shown to be the only root.]  9709/31/O/N/09

Question 59:
Solve the inequality $|x + 3a| > 2|x - 2a|$, where $a$ is a positive constant.  [4]

Answers:

**EITHER:** State or imply non-modular inequality $(x + 3a)^2 > (2(x - 2a))^2$, or corresponding quadratic equation, or pair of linear equations $(x + 3a) = \pm 2(x - 2a)$  B1

Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations  M1

Obtain critical values $x = \frac{1}{3}a$ and $x = 7a$  A1

State answer $\frac{1}{3}a < x < 7a$  A1

**OR:** Obtain the critical value $x = 7a$ from a graphical method, or by inspection, or by solving a linear equation or inequality  B1

Obtain the critical value $x = \frac{1}{3}a$ similarly  B2

State answer $\frac{1}{3}a < x < 7a$  B1

[Do not condone $\leq$ for $<$; accept 0.33 for $\frac{1}{3}$]  9709/31/M/J/10

Question 60:
Solve the equation

$$\frac{2^x + 1}{2^x - 1} = 5,$$

giving your answer correct to 3 significant figures.  [4]

Answers:

**EITHER:** Attempt to solve for $2^x$  M1

Obtain $2x = 6/4$, or equivalent  A1

Use correct method for solving an equation of the form $2^x = a$, where $a > 0$  M1

Obtain answer $x = 0.585$  A1
Question 61:
Solve the inequality $|x - 3| > 2|x + 1|$.  

**Answers:**

Either:

State or imply non-modular inequality $(x - 3)^2 > (2(x + 1))^2$, or corresponding quadratic equation, or pair of linear equations $(x - 3) = \pm 2(x + 1)$  B1

Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations  M1

Obtain critical values $-5$ and $\frac{1}{3}$  A1

State answer $-5 < x < \frac{1}{3}$  A1

Or:

Obtain the critical value $x = -5$ from a graphical method, or by inspection,  B1

Obtain the critical value $x = \frac{1}{3}$ similarly  B2

State answer $-5 < x < \frac{1}{3}$  B1

[Do not condone ≤ for <; accept 0.33 for $\frac{1}{3}$.]  

9709/32/M/J/10

Question 62:
Solve the inequality $2|x - 3| > |3x + 1|$.  

**Answers:**

Either:

State or imply non-modular inequality $(2(x - 3))^2 > (3x + 1)^2$, or corresponding quadratic equation, or pair of linear equations $2(x - 3) = \pm (3x + 1)$  B1

Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations  M1

Obtain critical values $x = -7$ and $x = 1$  A1

State answer $-7 < x < 1$  A1

Or:

Obtain critical value $x = -7$ or $x = 1$ from a graphical method, or by inspection,  B1

Obtain critical values $x = -7$ and $x = 1$  B2

State answer $-7 < x < 1$  B1

[Do not condone: < for <.]  

9709/33/M/J/10

Question 63:
Solve the inequality $|x| < |5 + 2x|$.  

**Answers:**

Either:

State or imply non-modular inequality $x^2 < (5 + 2x)^2$, or corresponding equation, or pair of linear equations $x = \pm (5 + 2x)$  M1

Obtain critical values $-5$ and $-\frac{5}{3}$ only  A1

Obtain final answer $x < -5$, $x > -\frac{5}{3}$  A1
Question 64:
Solve the equation \(|4 - 2^x| = 10\), giving your answer correct to 3 significant figures. [3]

Answer:
State or imply \(4 - 2^x = -10\) and 10 \(B1\)
Use correct method for solving equation of form \(2^x = a\) \(M1\)
Obtain 3.81 \(A1\) [9709/31/M/J/12]

Question 65:
Find the set of values of \(x\) satisfying the inequality \(3|x - 1| < |2x + 1|\). [4]

Answers:
EITHER State or imply non-modular inequality \((3(x - 1))^2 < (2x + 1)^2\) \(B1\)
or corresponding quadratic equation, or pair of linear equations \(3(x - 1) = \pm (2x + 1)\) \(M1\)
Make reasonable solution attempt at a 3-term quadratic, or solve two linear

equations
Obtain critical values \(x = \frac{2}{5}\) and \(x = 4\) \(A1\)
State answer \(\frac{2}{5} < x < 4\) \(A1\)

OR Obtain critical value \(x = \frac{2}{5}\) or \(x = 4\) from a graphical method, or by inspection, or by
solving a linear equation or inequality \(B1\)
Obtain critical values \(x = \frac{2}{5}\) and \(x = 4\) \(B2\)
State answer \(\frac{2}{5} < x < 4\) \(B1\)
[Do not condone \(\leq\) or \(\geq\)] [9709/32/O/N/12]

Question 66:
Solve the equation \(5^{x-1} = 5^x - 5\), giving your answer correct to 3 significant figures. [4]

Answer:
EITHER Use laws of indices correctly and solve for \(5^x\) or for \(5^{-x}\) or for \(5^{x-1}\) \(M1\)
Obtain \(5^x\) or \(5^{-x}\) or \(5^{x-1}\) in any correct form, e.g. \(5^x = \frac{5}{1 - \sqrt{5}}\) \(A1\)
Use correct method for solving \(5^x = a\), or \(5^{-x} = a\), or \(5^{x-1} = a\), where \(a > 0\) \(M1\)
Obtain answer \(x = 1.14\) \(A1\)

OR Use an appropriate iterative formula, e.g. \(x_{n+1} = \frac{\ln(5^{x-1} + 5)}{\ln 5}\), correctly, at least once \(M1\)
Obtain answer 1.14 \(A1\)
Show sufficient iterations to at least 3 d.p. to justify 1.14 to 2 d.p., or show there is a sign change in the interval (1.135, 1.145) \(A1\)
Show there is no other root \(A1\)
[For the solution \(x = 1.14\) with no relevant working give \(B1\), and a further \(B1\) if \(1.14\) is shown to be the only solution.] [9709/32/O/N/12]

Question 67:
(i) Solve the equation \(|4x - 1| = |x - 3|\). [3]

(ii) Hence solve the equation \(|4^{x+1} - 1| = |4^x - 3|\) correct to 3 significant figures. [3]
Answers:

(i) Either State or imply non-modular equation $(4x-1)^2 = (x-3)^2$ or pair of linear equations $4x-1 = \pm (x-3)$ B1

Solve a three-term quadratic equation or two linear equations M1

Obtain $\frac{2}{3}$ and $\frac{4}{5}$ A1

Or Obtain value $\frac{2}{3}$ from inspection or solving linear equation B1

Obtain value $\frac{4}{5}$ similarly B2

(ii) State or imply at least $4^\frac{1}{4} = \frac{4}{5}$, following a positive answer from part (i) B1√

Apply logarithms and use $\log a^b = b \log a$ property M1

Obtain $-0.161$ and no other answer A1 9709/31/M/J/13

Question 68:
Solve the equation $2 \mid 3^x - 1 \mid = 3^x$, giving your answers correct to 3 significant figures. [4]

Answers:

EITHER: State or imply non-modular equation $2^2 (3^x - 1)^2 = (3^x)^2$, or pair of equations

$2 \left(3^x - 1\right)^2 = \pm 3^x$ M1

Obtain $3^x = 2$ and $3^x = \frac{2}{3}$ (or $3^x + 1 = 2$) A1

OR: Obtain $3^x = 2$ by solving an equation or by inspection B1

Obtain $3^x = \frac{2}{3}$ (or $3^x + 1 = 2$) by solving an equation or by inspection B1

Use correct method for solving an equation of the form $3^x = a$ (or $3^x + 1 = a$), where $a > 0$ M1

Obtain final answers 0.631 and -0.369 A1 9709/32/O/N/13

Question 69:
Solve the inequality $x^2 - x - 2 > 0$. [3]

Answer:

$(x + 1)(x - 2)$ or other valid method M1

$-1, 2$ A1

$x < -1, x > 2$ A1

Attempt soln of eqn or other method

Penalise $\leq, \geq$ [3] 9709/13/O/N/13

Question 70:
Find the set of values of $x$ satisfying the inequality $|x + 2a| > 3|x - a|$, where $a$ is a positive constant. [4]

Answers:

EITHER: State or imply non-modular inequality $(x + 2a)^2 > (3(x-a))^2$, or corresponding quadratic equation, or pair of linear equations $(x + 2a) = \pm 3(x - a)$ B1

Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations for $x$ M1

Obtain critical values $x = \frac{1}{4}a$ and $x = \frac{5}{2}a$ A1

State answer $\frac{1}{4}a < x < \frac{5}{2}a$ A1
Question 71:
(i) Express $4x^2 - 12x$ in the form $(2x + a)^2 + b$. [2]
(ii) Hence, or otherwise, find the set of values of $x$ satisfying $4x^2 - 12x > 7$. [2]

Answers:

(i) $(2x-3)^2 - 9$

(ii) $2x - 3 > 4 \quad 2x - 3 < -4$

$x > \frac{3}{2}$ or $x < -\frac{1}{2}$

Allow $\frac{3}{2} > x > 3 \frac{1}{2}$

OR

$4x^2 - 12x - 7 \to (2x - 7)(2x + 1)$

$x > \frac{3}{2}$ or $x < -\frac{1}{2}$

Allow $\frac{3}{2} > x > 3 \frac{1}{2}$

Question 72:
Solve the inequality $|3x - 1| < |2x + 5|$. [4]

Answers:

Either

- State or imply non-modular inequality $(3x - 1)^2 < (2x + 5)^2$ or corresponding quadratic equation or pair of linear equations $3x - 1 = \pm (2x + 5)$ B1
- Solve a three-term quadratic or two linear equations $5x^2 - 26x - 24 < 0$ M1
- Obtain $\frac{4}{5}$ and 6 A1
- State $\frac{4}{5} < x < 6$ A1

Or

- Obtain value 6 from graph, inspection or solving linear equation B1
- Obtain value $\frac{4}{5}$ similarly B2
- State $\frac{4}{5} < x < 6$ B1

Question 73:
Using the substitution $u = 4^x$, solve the equation $4^x + 4^2 = 4^{x+2}$, giving your answer correct to 3 significant figures. [4]

Answer:

- Use laws of indices correctly and solve for $u$ M1
- Obtain $u$ in any correct form, e.g. $u = \frac{16}{16 - 1}$ A1
- Use correct method for solving an equation of the form $4^x = a$, where $a > 0$ M1
- Obtain answer $x = 0.0466$ A1

Question 74:
Solve the inequality $|x - 2| > 2x - 3$. [4]
Answers:

**Question 75:**
Solve the inequality $|2x - 5| > 3|2x + 1|$.  

**Answers:**

Either: State or imply non-modular inequality $(2x-5)^2 > (3(2x+1))^2$, or corresponding quadratic equation, or pair of linear equations $(2x-5) = \pm 3(2x+1)$. Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations for $x$. Obtain critical values $-2$ and $\frac{1}{4}$. State final answer $-2 < x < \frac{1}{4}$. 

**Question 76:**
Using the substitution $u = 3^x$, solve the equation $3^x + 3^{2x} = 3^{3x}$ giving your answer correct to 3 significant figures.  

**Answer:**

State or imply $1 + u = u^2$. Solve for $u$. Obtain root $\frac{1}{2}(1 + \sqrt{5})$, or decimal in $[1.61, 1.62]$. Use correct method for finding $x$ from a positive root. Obtain $x = 0.438$ and no other answer.  

**Question 77:**
(i) Solve the equation $2|x - 1| = 3|x|$.  
(ii) Hence solve the equation $2|5^x - 1| = 3|5^x|$, giving your answer correct to 3 significant figures.
Answers:

(i) EITHER: State or imply non-modular equation \((2(x - 1))^2 = (3x)^2\), or pair of linear equations

\[2(x-1) = \pm 3x\]

Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations

Obtain answers \(x = -2\) and \(x = \frac{1}{2}\)

OR: Obtain answer \(x = -2\) by inspection or by solving a linear equation

Obtain answer \(x = \frac{1}{2}\) similarly

(ii) Use correct method for solving an equation of the form \(5^x = a\) or \(5^{x+1} = a\), where \(a > 0\)

Obtain answer \(x = -0.569\) only

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Question 78:
Solve the inequality \(2|x - 2| > |3x + 1|\).

**Answers:**

EITHER: State or imply non-modular inequality \((2(x - 2))^2 > (3x + 1)^2\), or corresponding quadratic equation, or pair of linear equations \(2(x - 2) = \pm (3x + 1)\)

Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations for \(x\)

Obtain critical values \(x = -5\) and \(x = \frac{1}{3}\)

State final answer \(-5 < x < \frac{1}{3}\)

OR: Obtain critical value \(x = -5\) from a graphical method, or by inspection, or by solving a linear equation or inequality

Obtain critical value \(x = \frac{1}{3}\) similarly

State final answer \(-5 < x < \frac{1}{3}\)

[Do not condone \(\leq\) for \(<\)]

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Question 79:
(i) Express \(x^2 + 6x + 2\) in the form \((x + a)^2 + b\), where \(a\) and \(b\) are constants.

(ii) Hence, or otherwise, find the set of values of \(x\) for which \(x^2 + 6x + 2 > 9\).

**Answers:**

(i) \((x + 3)^2 - 7\)

(ii) \(1\) seen

\(x > 1\) or \(x < -7\)

[Allow \(x \leq -7, x \geq 1\) oe]

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Question 80:
Solve the equation \(\frac{3^x + 2}{3^x - 2} = 8\), giving your answer correct to 3 decimal places.

**Answers:**

Solve for \(3^x\) and obtain \(3^x = \frac{5}{2}\)

Use correct method for solving an equation of the form \(3^x = a\), where \(a > 0\)

Obtain answer \(x = 0.860\) 3 d.p. only

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Question 81:
Solve the inequality \(|x - 3| < 3x - 4\).

**Answers:**
Question 82:
Using the substitution \( u = e^x \), solve the equation \( 4e^{-x} = 3e^x + 4 \). Give your answer correct to 3 significant figures.

Answer:

\[
\text{Rearrange as } 3u^2 + 4u - 4 = 0, \quad \text{or } 3e^{2x} + 4e^x - 4 = 0, \quad \text{or equivalent } \quad B1
\]

\[
\text{Solve a 3-term quadratic for } e^x \text{ or for } u \quad M1
\]

\[
\text{Obtain } e^x = \frac{2}{3} \text{ or } u = \frac{2}{3} \quad A1
\]

\[
\text{Obtain answer } x = -0.405 \text{ and no other } \quad A1
\]

<table>
<thead>
<tr>
<th>Question 82:</th>
</tr>
</thead>
<tbody>
<tr>
<td>It is given that the variable ( x ) is such that</td>
</tr>
<tr>
<td>( 1.3^{2x} &lt; 80 \quad \text{and} \quad</td>
</tr>
<tr>
<td>Find the set of possible values of ( x ), giving your answer in the form ( a &lt; x &lt; b ) where the constants ( a ) and ( b ) are correct to 3 significant figures.</td>
</tr>
<tr>
<td><strong>Answers:</strong></td>
</tr>
</tbody>
</table>

Total: 4
Question 83:
Solve the inequality $|3x - 2| < |x + 5|$.

Answer:

**Either**

State or imply non-modular inequality $(3x - 2)^2 < (x + 5)^2$ or corresponding equation or pair of linear equations  

Attempt solution of 3-term quadratic equation or of 2 linear equations  

Obtain critical values $-\frac{3}{4}$ and $\frac{7}{2}$  

State answer $-\frac{3}{4} < x < \frac{7}{2}$  

Or

Obtain critical value $\frac{7}{2}$ from graph, inspection, equation  

Obtain critical value $-\frac{3}{4}$ similarly  

State answer $-\frac{3}{4} < x < \frac{7}{2}$

Question 84:
Showing all necessary working, solve the equation $3|2^x - 1| = 2^x$, giving your answers correct to 3 significant figures.

Answers:
**Question 85:**
Showing all necessary working, solve the equation $5^{2x} = 5^x + 5$. Give your answer correct to 3 decimal places.

**Answer:**

State or imply $u^2 = u + 5$, or equivalent in $5^x$.  

Solve for $u$, or $5^x$.  

Obtain root $\frac{1}{2}(1 + \sqrt{21})$, or decimal in [2.79, 2.80].  

Use correct method for finding $x$ from a positive root.  

Obtain answer $x = 0.638$ and no other answer.

---

**Question 86:**
Solve the inequality $|3x - 5| < 2|x|$.  

**Answers:**
### Question 87:

Find the set of values of $x$ satisfying the inequality $2|2x - a| < |x + 3a|$, where $a$ is a positive constant. [4]

**Answers:**

**EITHER:** State or imply non-modular inequality $2^2(2x - a)^2 < (x + 3a)^2$, or corresponding quadratic equation, or pair of linear equations $2(2x - a) = \pm (x + 3a)$

<table>
<thead>
<tr>
<th>State or imply non-modular inequality</th>
<th>B1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2^2(2x - a)^2 &lt; (x + 3a)^2$</td>
<td></td>
</tr>
<tr>
<td>or corresponding quadratic equation, or pair of linear equations $2(2x - a) = \pm (x + 3a)$</td>
<td></td>
</tr>
</tbody>
</table>

Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for $x$

<table>
<thead>
<tr>
<th>Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for $x$</th>
<th>M1</th>
</tr>
</thead>
</table>

Obtain critical values $x = \frac{-5}{3}a$ and $x = -\frac{1}{3}a$

<table>
<thead>
<tr>
<th>Obtain critical values $x = \frac{-5}{3}a$ and $x = -\frac{1}{3}a$</th>
<th>A1</th>
</tr>
</thead>
</table>

State final answer $-\frac{1}{5}a < x < \frac{5}{3}a$

<table>
<thead>
<tr>
<th>State final answer $-\frac{1}{5}a &lt; x &lt; \frac{5}{3}a$</th>
<th>A1</th>
</tr>
</thead>
</table>

**OR:** Obtain critical value $x = \frac{5}{3}a$ from a graphical method, or by inspection, or by solving a linear equation or an inequality

<table>
<thead>
<tr>
<th>Obtain critical value $x = \frac{5}{3}a$ similarly</th>
<th>B2</th>
</tr>
</thead>
</table>

State final answer $-\frac{1}{5}a < x < \frac{5}{3}a$

<table>
<thead>
<tr>
<th>State final answer $-\frac{1}{5}a &lt; x &lt; \frac{5}{3}a$</th>
<th>B1</th>
</tr>
</thead>
</table>

[Do not condone $\leq$ for $<$ in the final answer.]

### Question 88:

Showing all necessary working, solve the equation $\frac{2e^x + e^{-x}}{e^x - e^{-x}} = 4$, giving your answer correct to 2 decimal places.

**Answer:**
Question 89:

(i) Solve the inequality $|3x - 5| < |x + 3|$.

(ii) Hence find the greatest integer $n$ satisfying the inequality $|3^{0.1n+1} - 5| < 3^{0.1n} + 3$.

Answers:

(i) State or imply non-modular inequality $(3x - 5)^2 < (x + 3)^2$ or corresponding equation or pair of different linear equations/inequalities

(ii) Attempt to find $n$ (not necessarily an integer so far) from $3^{0.1n} = -5$ or $3^{0.1n+1} = 3 \times 3^{0.1n}$ or equivalent

Conclude 12

Question 90:

(i) Solve the equation $|4 + 2x| = |3 - 5x|$.

(ii) Hence solve the equation $|4 + 2e^{3x}| = |3 - 5e^{3x}|$, giving the answer correct to 3 significant figures.

Answers:

(i) State or imply non-modular equation $(4 + 2x)^2 = (3 - 5x)^2$ or pair of linear equations

(ii) Attempt correct process to solve $e^{3x} = k$ where $k > 0$ from (i)

Obtain 0.282 and no others

Question 91:

Showing all necessary working, solve the equation $9^x = 3^x + 12$. Give your answer correct to 2 decimal places.

Answer:
Question 92:
(i) Solve the inequality $|2x - 7| < |2x - 9|$. 
(ii) Hence find the largest integer $n$ satisfying the inequality $|2\ln n - 7| < |2\ln n - 9|$. 

**Answer:**
(i) State or imply non-modular inequality $(2x - 7)^2 < (2x - 9)^2$ or corresponding equation or linear equation (with signs of $2x$ different)

Obtain critical value $4$

State $x < 4$ only

(ii) Attempt to find $n$ from $\ln n = their$ critical value from part (i)

Obtain or imply $n < e^4$ and hence $54$

---

Question 93:
(i) Solve the equation $|4x + 5| = |x - 7|$. 
(ii) Hence, using logarithms, solve the equation $|2^{x+2} + 5| = |2^x - 7|$, giving the answer correct to 3 significant figures.

**Answer:**
(i) State or imply non-modular equation $(4x + 5)^2 = (x - 7)^2$ or pair of different linear equations

Attempt solution of 3-term quadratic equation or pair of linear equations

Obtain $\frac{1}{2}$ and $-4$

(ii) Apply logarithms and use power law for $2^x = k$ where $k > 0$ from (i)

Obtain $-1.32$ only

---

Question 94:
Solve the inequality $|2x - 3| > 4|x + 1|$. 

**Answer:**
### Question 95:
Solve the inequality $2|x + 2| > |3x - 1|$.  

**Answer:**

<table>
<thead>
<tr>
<th>State or imply non-modular inequality</th>
<th>B1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(2x – 3)^2 &gt; 4(3x - 1)^2$, or corresponding quadratic equation, or pair of linear equations</td>
<td></td>
</tr>
</tbody>
</table>

| Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for $x$ | M1 |
| Correct method seen, or implied by correct answers |    |

| Obtain critical values $x = -\frac{7}{2}$ and $x = -\frac{1}{6}$ | A1 |
| State final answer $-\frac{7}{2} < x < -\frac{1}{6}$ | A1 |

#### Alternative method for question 2

| Obtain critical value $x = -\frac{7}{2}$ from a graphical method, or by inspection, or by solving a linear equation or an inequality | B1 |
| Obtain critical value $x = -\frac{1}{6}$ similarly | B2 |

| State final answer $-\frac{7}{2} < x < -\frac{1}{6}$ | B1 |

---

### Question 96:

(i) On the axes below, sketch the graph of $y = |2x^2 - 9x - 5|$ showing the coordinates of the points where the graph meets the axes.  

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(ii) Find the values of $k$ for which $|2x^2 - 9x - 5| = k$ has exactly 2 solutions. [3]

**Answer:**

<table>
<thead>
<tr>
<th>Question 97:</th>
<th>![Diagram]</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) On the axes below, sketch the graph of $y =</td>
<td>2x + 5</td>
</tr>
</tbody>
</table>
(b) Solve \(|2x + 5| \leq |2 - x|\). [3]

**Answers:**

<table>
<thead>
<tr>
<th>(a)</th>
<th>(b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correct shape for both graphs</td>
<td>(2x + 5 = \pm(2 - x)) or ((2x + 5)^2 = (2 - x)^2)</td>
</tr>
<tr>
<td>Correct (y)-intercept for both graphs and (x)-intercept for both graphs</td>
<td>(x = -7, x = -1)</td>
</tr>
<tr>
<td>B2</td>
<td>A1</td>
</tr>
<tr>
<td>B2</td>
<td>A1</td>
</tr>
<tr>
<td>B1 for either (y = 2), (y = 5) or (x = 2, x = -2.5) or (y = 2, x = -2.5)</td>
<td>FT their values of (x)</td>
</tr>
</tbody>
</table>

**Question 98:**

(a) On the axes below, sketch the graph of \(y = \frac{1}{5}(x - 2)(x - 4)(x + 5)\), showing the coordinates of the points where the graph meets the coordinate axes.
(b) Explain why your sketch in part (a) can be used to solve \((x - 2)(x - 4)(x + 5) \leq 0\). 

(c) Hence solve \((x - 2)(x - 4)(x + 5) \leq 0\).

**Answers:**

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>(a)</td>
<td><img src="image" alt="Graph" /></td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B1 for shape</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B1 for intercepts on coordinate axes</td>
</tr>
</tbody>
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<tr>
<th></th>
<th></th>
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<tbody>
<tr>
<td>(b)</td>
<td>Valid explanation, e.g. multiplying throughout by 5 does not change x values because the x-axis is invariant.</td>
<td>1</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
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<th></th>
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</thead>
</table>
| (c) | \(x \leq -5\)  
\(2 \leq x \leq 4\) | 1 |
|   | FT their graph |   |